Public Liquidity and Intermediated Liquidity*

Ye Li[†]

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Abstract

Under financial constraints, firms hold liquid assets in anticipation of investment needs. Financial intermediaries supply liquid securities in the form their short-term debts. Endogenous liquidity creation stimulates investment and economic growth but generates intermediary leverage cycle that destabilizes the economy. Introducing government debt as an alternative source of liquidity is a double-edged sword. On the one hand, firms hold more liquidity in every state of the economy. On the other hand, by crowding out intermediaries' profits from liquidity provision, government debt induces intermediaries to reach for yield via procyclical risk-taking. As a result, the stationary (long-run) probability distribution tilts towards crisis states where intermediaries are undercapitalized and supply less liquidity. The latter force dominates, when public liquidity cannot satiate firms' liquidity demand and intermediaries are still the marginal liquidity suppliers. Contrary to the literature, this paper demonstrates an overall negative impact of public liquidity on both long-run growth and financial stability.

Keywords: Procyclicality, government debt, liquidity, leverage cycle, reaching for yield

1   Introduction

In the decades leading up to the Great Recession, the financial sector grew rapidly, setting a favorable liquidity condition that stimulated the real economy. A booming real sector in turn fueled financial intermediaries' expansion and leverage. During the crisis, the spiral flipped. Through the boom-bust cycle, public debt has risen to a historically high level. Much progress has been made to incorporate intermediaries in macro models, yet a complete account of procyclical intermediation remains a challenge and the role of public debt in the intermediation cycle is not understood.

This paper argues that at the heart of this procyclicality is the liquidity-provision function of intermediaries' debt, and that government debt, as a competing source of liquidity, amplifies such procyclicality. I build a continuous-time model of macroeconomy where firms hold liquid assets to finance unpledgeable investment needs (as in Holmström and Tirole (1998)) and banks supply such assets by issuing debt, building up leverage in the process. This setup is motivated by the fact that nonfinancial corporate sector has become a net saver (Quadrini, 2017) and a major cash pool that lends to financial intermediaries (Greenwood, Hanson, and Stein, 2016). This investment-driven liquidity demand differs from the traditional liquidity demand from households (typically modelled through money-in-utility), and such difference has unique implications on bank leverage dynamics. Specifically, firms' liquidity demand exhibits intertemporal complementarity in a dynamic setting, which generates procyclicality in bank leverage and persistent effects of shocks to bank balance-sheet capacity. Bank leverage cycle in turn leads to fragile booms and stagnant crises.

The idea that banks affect the real economy through liquidity ("inside money") supply goes back at least to Friedman and Schwartz (1963).[1] One may argue that this money view of banking is less relevant today given the seemingly ample "outside money" in the form of central bank and government liabilities (Woodford, 2010). However, the model shows that competition between inside and outside money *destabilizes* the banking sector by amplifying its leverage cycle. These results complement the recent literature on outside money as a means to financial *stability* (Greenwood, Hanson, and Stein, 2015; Krishnamurthy and Vissing-Jørgensen, 2015; Woodford, 2016).

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[1]The term "inside money" is from Gurley and Shaw (1960). From the private sector's perspective, central bank and government liabilities are in positive supply (outside money), while bank debt is in zero net supply (inside money).

The model is first set up without a government, generating the benchmark dynamics, and then government debt is introduced and its impact analyzed. There are three types of agents: bankers, entrepreneurs ("firms"), and households who play a limited role. Agents are risk-neutral with the same time discount rate, and consume non-durable generic goods produced by firms' capital.

Firms can hold capital and bank deposits as assets, and borrow from banks and households *as long as* they are not hit by liquidity shocks. Every instant, firms face a constant probability of idiosyncratic liquidity shock, and in such an event, their production halts, and their capital can either grow – if further investment is made – or perish, if not. This investment is not pledgeable, so firms can only obtain goods (investment inputs) from others in spot transactions, using deposits as means of payment. Banks add value because their debts are held by firms as liquidity buffers.

Bankers issue deposits that are short-term safe debts, and extend loans to firms that are backed by designated capital as collateral. Every instant, a random fraction of capital collateral is destroyed, and the corresponding loans default. The only aggregate shock is a Brownian motion that drives this random destruction of capital. A negative shock means more capital destroyed and larger loss in bankers' loan portfolio. In sum, every instant, firms face two types of events – the idiosyncratic liquidity shock, which necessitates investment paid by deposits, and the random destruction of their existing capital that loads on aggregate shock. These two events are independent.

Liquidity creation requires risk-taking. At the margin, one more dollar of safe deposits is backed by one more dollar of risky loan. Therefore, liquidity supply depends on bank equity as risk buffer. Bankers may raise equity subject to an issuance cost. This friction ties liquidity supply to the current level of bank equity. When bad shocks leave more loans in default and deplete bank equity, liquidity supply declines, which hurts firms' liquidity management and investment.

The model has a Markov equilibrium with the ratio of bank equity to firm capital as state variable, which is intuitively the size of liquidity suppliers relative to that of liquidity demanders. Because banks have leveraged exposure to the capital destruction shock, this state variable rises following good shocks, and falls following bad shocks. It is also bounded by two endogenous boundaries: when the banking sector is small, bankers raise equity because the marginal value of equity is sufficiently high to justify costly equity issuance; when the banking sector is large, the

marginal value of equity falls to one, so bankers consume and pay out dividends to shareholders.[2]

Bank leverage is procyclical.[3] Good shocks increase bank equity, but banks issue even more debt to meet firms' procyclical liquidity demand. After good shocks, banks hoard loan profits instead of paying it out because the equity issuance cost creates a precautionary savings motive, reflected in a wedge between the marginal value of retained equity and one dollar (the payout value). Thus, shock impact is persistent, dissipating gradually. Expecting banks to be better capitalized and to supply more liquidity going forward, firms foresee themselves to carry more liquidity *in the future* that will then finance faster capital growth. Thereby, capital becomes more valuable through the improved growth options, inducing firms to hold more liquidity *now* in case the liquidity shock arrives the very next instant and investment is needed to maintain and grow capital.

In sum, firms' liquidity demand exhibits intertemporal complementarity: Following good shocks, firms' expectation of more liquidity supply and faster capital growth in the future drives up their current savings in the form of bank debt that finance any immediate investment needs, and as firms' savings grow, banks are able to issue debt faster than they build up equity, increasing leverage. Following bad shocks, firms' expectation of less liquidity and slower capital growth in the future discourages them from saving for investment now, accelerating banks deleveraging. Here asset price (i.e., capital value or Tobin's Q) plays a key role, feeding the expectation of future liquidity conditions into firms' current liquidity demand. Note that such mechanism is absent from the standard money-in-utility specification that aims to capture households' liquidity needs. The intertemporal complementarity in firms' liquidity demand contributes to the procyclicality of bank leverage. Endogenous asset price has a unique destabilizing effect that is distinct from the typical balance-sheet channel (e.g., Brunnermeier and Sannikov (2014)).

Downside risk accumulates through procyclical leverage. Following good shocks, as banks become more levered, their equity is more sensitive to shocks. And, as the economy approaches bank payout boundary, high leverage only serves to amplify bad shocks, because good shocks cannot increase bank equity above the boundary without triggering payout. The longer booms last (i.e., closer to the bank payout boundary), the greater downside risk is.

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[2]The payout and issuance policies are consistent with the evidence in Adrian, Boyarchenko, and Shin (2015).

[3]The leverage here is the ratio of book asset to book equity, as will be clearly defined in the model.

Crises are stagnant. As the economy approaches bank issuance boundary, low bank leverage only serves to dampen good shocks, because bank equity never falls below the issuance boundary. Therefore, banks can only rebuild equity after long periods of good shocks. The calibrated model produces an eight-year recovery period, during which the economy is stuck with insufficient liquidity supply that compromises firms' liquidity management. In sum, fragile booms and stagnant crises result from a combination of procyclical leverage and the asymmetric impact of shocks near the reflecting boundaries endogenously determined by bank payout and equity issuance decisions.

So far, we have focused on the procyclical quantity of liquidity. The model also generates a countercyclical price of liquidity, the *liquidity premium*, which is a spread between the intertemporal discount rate and deposit rate. Paying an interest rate lower than the time discount rate, banks earn the liquidity premium. Carrying low-yield deposits, firms pay the liquidity premium, which is a cost of liquidity management, but firms optimally do so in anticipation of investment needs.

Next, I introduce government debt as an alternative source of liquidity. To highlight the competition between intermediated and public liquidity, I abstract away other distortions by assuming the debt issuance proceeds are paid to agents as lump-sum transfer and debt is repaid with lump-sum tax. Government debt decreases the liquidity premium in every state of the world, which seems to indicate a more favorable condition for firms and more investment and economic growth as a result. However, the general-equilibrium impact depends on how banks respond.

By lowering the liquidity premium, government debt increases banks' debt cost, and thereby, decreases bank profits in every state of the world. However, in equilibrium, banks must maintain a sufficiently high level of profits over the long run to offset the occasionally incurred costs of equity issuance. With bank profits lower in every state of the world, the stationary (long-run) probability distribution must shift towards states where the liquidity premium and bank profits are relatively high (i.e, the bad states with less liquidity) to sustain banks' profits over the long run. Such shift of probability mass can only happen if banks' leverage becomes more procyclical. A higher leverage in good states makes bank equity and, importantly, the state variable, more sensitive to shocks, so the economy spends less time in good states. By the same logic, a lower leverage in bad states makes the economy less sensitive to shocks and stuck in these states.

Therefore, in response to the profit crowding-out effect of government debt issuance, the banking sector optimally sets a more procyclical leverage, which reflects a form of reaching for yield. In good states where banks are well capitalized and willing to take risks, they raise leverage even higher, so that on average, their return on equity (i.e., the net interest margin multiplied by leverage) is still high enough to justify the occasionally incurred cost of equity issuance. In the transition from good to bad states following negative shocks, banks deleverage faster, quickly retreating into a "defensive" mode to avoid further depletion of equity and costly equity issuance.

With booms being more fragile and crises more stagnant, the economy spends more time in states where banks are undercapitalized and their supply of liquidity is low. In those states, firms pay a high liquidity premium and hold less liquidity for investment. Unless public liquidity satiates firms' liquidity demand, the economy still relies on banks as the marginal suppliers of liquidity. Therefore, even if government debt increases the total liquidity supply in *every* state of the world, by amplifying the procyclicality of bank leverage and increasing the likelihood of the states with relatively less liquidity supplied by banks, public liquidity can result in a lower long-run *average* supply of liquidity, and thereby, reduce welfare. The impact of government debt on bank leverage cycle and the liquidity crowding-out effect are unique predictions of the model. From an empirical pointview, the model implies a reduction of liquidity premium in the short term in response to government debt issuance (i.e., a decrease of liquidity premium in the current state of the world), but over the long run, the average liquidity premium increases.

Related literature. The liquidity-provision function of bank liabilities has received enormous attention recently. Agents hold bank liabilities as precautionary savings when markets are incomplete (Brunnermeier and Sannikov, 2016; Quadrini, 2017) or as means of payment under limited commitment (Bigio and Weill, 2016; Hart and Zingales, 2014).[4] This paper advances this line of research in three aspects. The first contribution is to show that firms' dynamic liquidity manage-

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[4]Several branches of literature provide microfoundations for bank debts serving as means of payment. Limited commitment (Kiyotaki and Moore, 2002) and imperfect record keeping (Kocherlakota, 1998) limits credit, so trades involve a settlement medium that is supplied by banks with superior commitment technology (Kiyotaki and Moore, 2000; Cavalcanti and Wallace, 1999). Another approach relates resalability to information sensitivity. Banks create money by issuing safe claims that circulate in secondary markets (Gorton and Pennacchi, 1990).

ment generates procyclicality in bank leverage.[5] Household ownership of money-like claims has been fairly stable, but corporate holdings of such claims have increased substantially since early 1990s (Greenwood, Hanson, and Stein, 2016). Motivated by this trend, this paper models firms' liquidity management as the source of demand for bank liabilities instead of households' money in utility (Stein, 2012).[6] Second, by modeling banks' dynamic balance-sheet management (i.e., leverage, payout, and equity issuance policies), this paper explains how the liquidity-provision function of bank liabilities is key to understand the frequency and duration of banking crisis. Third, this paper is the first to show how public liquidity affects bank leverage cycle.

This paper contributes to the literature on asset shortage under financial frictions (Caballero and Krishnamurthy, 2006; Caballero et al., 2008; Farhi and Tirole, 2012; Giglio and Severo, 2012; Kocherlakota, 2009; Martin and Ventura, 2012; Miao and Wang, 2018). My model has two unique features. First, the severity of asset (liquidity) shortage depends on banks' balance-sheet capacity. Second, through rational expectation of future liquidity supply, entrepreneurs' liquidity demand exhibits intertemporal complementarity that contributes to the procyclicality in bank leverage. These new features allow the model to show for the first time how asset shortage and bank leverage cycle are intertwined, and how introducing government debt to alleviate asset shortage may backfire by amplifying bank leverage cycle, and thereby, shortening booms and prolonging crises.

The liquidity service of government liabilities is an old theme (Patinkin, 1965; Friedman, 1969). Recent contributions include Bansal and Coleman (1996), Bansal, Coleman, and Lundblad (2011), Krishnamurthy and Vissing-Jørgensen (2012), Greenwood, Hanson, and Stein (2015), and Nagel (2016). This paper advances the research on the value of public liquidity under market incompleteness (Aiyagari and McGrattan, 1998; Azzimonti, de Francisco, and Quadrini, 2014) by emphasizing government as banks' competitor in liquidity supply. As in Woodford (1990b) and Holmström and Tirole (1998), government debt stimulates investment and growth by allowing en-

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[5]The existing models attribute leverage procyclicality to the connection between (countercyclical) uncertainty and leverage through various binding constraints motivated by risk shifting or collateral requirement (Brunnermeier and Pedersen, 2009; Geanakoplos, 2010; Adrian and Boyarchenko, 2012; Danielsson, Shin, and Zigrand, 2012; Moreira and Savov, 2014). Nuño and Thomas (2017) provide a quantitative assessment of this class of models.

[6]Eisfeldt (2007) shows that liquidity demand from consumption smoothing cannot explain the liquidity premium on Treasury bills. Eisfeldt and Rampini (2009) show that liquidity premium rises when asynchronicity between corporate cash flow and investment becomes more severe, consistent with the prediction of Holmström and Tirole (2001).

trepreneurs to buffer against uninsurable liquidity shocks. However, by crowding out intermediated liquidity, public liquidity can reduce the overall liquidity and welfare.

After the financial crisis, governments in advanced economies increased their indebtedness and central banks expanded balance sheets, raising concerns such as moral hazard in the financial sector and excessive inflation (Fischer, 2009). This paper highlights a financial instability channel through which an expanding government balance sheet can be counterproductive. Many have argued that government debt stabilizes banks by crowding out bank debt (Greenwood, Hanson, and Stein, 2015; Krishnamurthy and Vissing-Jørgensen, 2015; Woodford, 2016). In contrast, by modeling banks' *dynamic* balance-sheet management under equity issuance cost, this paper shows that government debt destabilizes the banking sector by amplifying its leverage cycle.

The equity issuance cost implies that shock impact is persistent, a "balance-sheet channel" in the model. Bank networth as financial slack is important, which is a feature shared with classic models of balance-sheet channel (Bernanke and Gertler, 1989). My model differs in two aspects. First, asset price (capital value) plays a role in shock amplification through the intertemporal complementarity of firms' liquidity demand, instead of the typical balance-sheet impact (e.g., Brunnermeier and Sannikov (2014)). Second, the demand for intermediary debt is dynamic, contributing to the procyclicality of bank leverage and endogenous risk accumulation. In contrast, existing models have a static/passive demand for intermediary debt that leads to countercyclical leverage (He and Krishnamurthy, 2013; Phelan, 2016; Klimenko, Pfeil, Rochet, and Nicolo, 2016).

This paper connects firms' liquidity demand and banks' liquidity supply. The existing models of corporate cash management ignore the supply side and assume a perfectly elastic supply of storage technology (Bolton, Chen, and Wang, 2011; Froot, Scharfstein, and Stein, 1993; He and Kondor, 2016; Riddick and Whited, 2009; Décamps, Mariotti, Rochet, and Villeneuve, 2011). The enormous amount of corporate cash holdings have received a lot of attention in the empirical literature (Bates, Kahle, and Stulz, 2009; Eisfeldt and Muir, 2016; Gao, Whited, and Zhang, 2018). Investment need is a key determinant of cash holdings (Denis and Sibilkov, 2010; Duchin, 2010), especially for firms with less collateral (Almeida and Campello, 2007) and more R&D activities

(Falato and Sim, 2014).[7] The setup of firms' liquidity shock is motivated by these findings.

The remainder is organized as follows. Section 2 analyzes the liquidity shortage in the private sector with a continuous-time formulation in Section 3. Section 4 analyzes the impact of public liquidity. Section 5 concludes. Appendices contain proofs, algorithm, and calibration details.

2 Static Model: An Anatomy of Private Liquidity Shortage

This section analyzes the inefficient supply of liquidity in the private sector in a two-period model ($t = 0, 1$). There are goods, capital, and three types of agents, households, bankers, and firms. Firms own capital that produces goods at $t = 1$, but some are hit by a liquidity shock before production and need to invest. To buffer the shock, firms carry deposits issued by bankers at $t = 0$. Bankers back deposits by loans extended to firms. Liquidity supply depends on bankers' balance-sheet capacity. Insufficient supply compromises firms' liquidity management and investment.

2.1 Setup

Physical structure. All agents consume a non-storable, generic good, and have the same risk-neutral utility with discount rate ρ. At $t = 0$, there are K_0 units of capital endowed to a unit mass of homogeneous entrepreneurs (firms). One unit of capital produces α units of goods at $t = 1$, and it is only productive in the hands of entrepreneurs. Capital can be traded in a competitive market at $t = 0$, at price q_0^K. Let k_0 denote a firm's holdings of capital, so that $K_0 = \int_{s \in [0,1]} k_0(s)\, ds$. I use subscripts for time, and whenever necessary, superscripts for type ("B" for bankers, "H" for households, and "K" for firms who own capital). There is also a unit mass of homogeneous bankers. Each is endowed with e_0 units of goods, so their aggregate endowment is $E_0 = \int_{s \in [0,1]} e_0(s)\, ds$. The index "$s$" will be suppressed without loss of clarity. There are a unit mass of homogeneous households endowed with a large amount of goods per period. Households play a very limited role.

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[7]The procyclicality of R&D expenditures, as measured by the NSF, has been documented by many studies, including Griliches (1990), Fatas (2000), and Comin and Gertler (2006). Using French firm-level data, Aghion et al. (2012) show that the procyclicality of R&D investment is found among firms that are financially constrained.

At the beginning of date 1 ($t = 1$), all firms experience a capital destruction shock, while some also experience a liquidity shock. The economy has one aggregate shock Z_1, a binary random variable that takes value 1 or -1 with equal probability. After Z_1 is realized, firms lose a fraction $\pi(Z_1)$ of their capital. For simplicity, I assume that $\pi(Z_1) = \delta - \sigma Z_1$ ($\delta - \sigma \geq 0$ and $\delta + \sigma \leq 1$). Later we will see Z_1 makes bank equity essential for liquidity creation. After capital loss, firms proceed to produce $\alpha[1 - \pi(Z_1)]k_0$ units of goods, *if* they not are hit by the liquidity shock.

Independent liquidity shocks hit firms with probability λ, and destroys *all* capital. In the spirit of Holmström and Tirole (1998) and (2001), firms must make further investment; otherwise, they can not produce anything, and thus, exit with zero terminal value. By investing i_1 units of goods per unit of capital, firms can create $F(i_1)k_0$ ($F'(\cdot) > 0$, $F''(\cdot) < 0$) units of new capital. Homogeneity in k_0 helps reduce the dimension of state variable later in the continuous-time dynamic analysis. I assume that after the investment, firms can revive their old capital, so the post-investment production is $\alpha[1 - \pi(Z_1) + F(i_1)]k_0$. Through investment, firms preserve the existing scale of production and grow. The first-best level of investment rate, i_{FB}, is defined by:

$$\alpha F'(i_{FB}) = 1, \tag{1}$$

Note that firms making investment at the beginning of $t = 1$ instead of $t = 0$. This backloaded specification gives rise to firms' liquidity demand later in the presence of financial constraints.

Last but not least, it is assumed that all securities issued by agents in this economy pay out *at the end of date 1*. This timing assumption is particularly relevant for defining what is liquidity from firms' perspective. Assets that firms carry to relax constraints on investment must be *resalable* in exchange for investment inputs *at the beginning of date 1*. Firms are not buy-and-hold investors. It will be shown that this resalability requirement relates the model setup to several strands of literature that study banks as issuers of inside money. Figure 1 shows how events unfold.

Liquidity demand. The model features three frictions, one that gives rise to firms' liquidity demand, and the other two limiting liquidity supply. The first friction is that investment has to be internally financed. In other words, the newly created capital is not pledgeable.[8] As a result, firms

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[8]This can be motivated by a typical moral hazard problem as in Holmström and Tirole (1998).

Date 0	Date 1			

Date 0

All agents
– Raise funds by issuing securities
– Consume
– Allocate savings

Date 1

Capital
– $\pi(Z_1)$ fraction destroyed

Investing Firms: λ
– Acquire goods as investment inputs
– Invest

Non-investing Firms: $1 - \lambda$
– Produce & sell goods

– Produce & sell goods

All agents
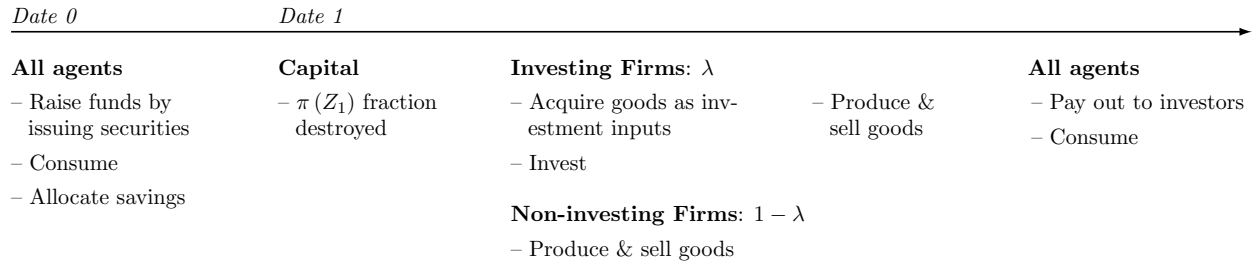– Pay out to investors
– Consume

Figure 1: **Timeline.** This figure plots the timeline of static model. Agents set balance sheets at $t = 0$. Z_1 is realized at the beginning of $t = 1$. $\pi(Z_t)$ fraction of capital is destroyed. Then the liquidity shock hits λ fraction of firms. The rest produce. λ firms produce after investments. All agents consume and repay liability holders at the end of $t = 1$.

need to carry liquidity (i.e., instruments that transfer wealth from $t = 0$ to $t = 1$). This is a common assumption used to model firms' liquidity demand.[9] To achieve the first-best investment, a firm must have access to liquidity of at least $i_{FB}k_0$ when the λ shock hits.

The objective of this paper is to analyze the *endogenous* supply of liquidity, so I assume that goods cannot be stored (i.e., there is no exogenous storage technology). And, capital cannot transfer wealth to a contingency where itself is destroyed without further spending. So firms must hold financial assets as liquidity buffer. While households receive endowments per period, it is assumed that they cannot sell claims on their future endowments because they can default with impunity; otherwise, there would be no liquidity shortage (as in Holmström and Tirole (1998)). Therefore, the focus is on the asset creation capacity of entrepreneurs themselves and bankers.

Liquidity supply. At date 0, what is a firm's capacity to issue claims that pay out at date 1? I assume firms' *endowed* capital is pledgeable.[10] It is collateral that can be seized by investors when default happens.[11] In an equilibrium where firms always carry liquidity and invest when hit by the λ shock, a fraction $[1 - \pi(Z_1)]$ of endowed capital is always preserved. Thus, a firm's pledgeable value at $t = 1$ is $\alpha(1 - \delta)k_0$ in expectation, and $\alpha(1 - \delta - \sigma)k_0$ when $Z_1 = -1$.

Potentially, firms could hold securities issued by each other as liquidity buffer. If the aggre-

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[9]See Froot, Scharfstein, and Stein (1993), Holmström and Tirole (1998) and (2001) in the micro literature, and Woodford (1990b), Kiyotaki and Moore (2005), and He and Kondor (2016) in the macro literature among others.

[10]New capital expected to be created at $t = 1$ is not pledgeable, in line with non-pledgeability of investment project.

[11]This reflects that mature capital can be relatively easily evaluated, verified, and seized by investors. Allowing capital created at date 1 to serve as collateral complicates the expressions but does not change the main results.

gate pledgeable value *always* exceeds firms' aggregate liquidity demand, i.e.,

$$\alpha \left(1 - \delta - \sigma \right) K_0 \geq i_{FB} K_0, \tag{2}$$

the economy achieves the first-best investment defined in Equation (1).[12] Even better, as long as the liquidity shock is verifiable, firms' liabilities can be pooled into a mutual fund that pays out to investing firms, so given this perfect risk-sharing, the first-best investment is achieved if

$$\alpha \left(1 - \delta - \sigma \right) K_0 \geq \lambda i_{FB} K_0, \text{ where } \lambda \in \left(0, 1 \right). \tag{3}$$

In a similar setting, Holmström and Tirole (1998) study the question whether entrepreneurs' supply of assets meets their own liquidity demand (i.e., Equation (2) and (3)), and emphasize the severity of liquidity shortage depends on aggregate shock (i.e., σ in my setting).

There are several reasons why firms hold *intermediated liquidity*. Entrepreneurs may simply lack the required expertise of asset management.[13] And, cross holding is regulated in many countries and industries. This assumption is also motivated by strands of theoretical literature that study banks as inside money creators. Given the timing in Figure 1, entrepreneurs purchase goods as investment inputs with their liquidity holdings when the λ shock hits. In other words, firms carry liquidity as a means of payment. Kiyotaki and Moore (2000) and Bigio and Weill (2016) model bankers as agents with superior ability to make multilateral commitment, i.e., to pay *whoever* holds their liabilities, so bank liabilities circulate as means of payment.[14] Moreover, liquidity creation may require not only specialists (i.e., bankers), but also a particular security design. In Gorton and Pennacchi (1990), banks create liquidity by issuing information-insensitive claims (safe debts)

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[12]Liquidity holdings cannot be pledged. Otherwise, pledgeable value is infinite: firms' issuance of securities enlarge each other's financing capacity, so more securities are issued. Holmström and Tirole (2011) make a related argument.

[13]Standard in the banking literature, banks pool risks, both on asset and liability sides (Diamond and Dybvig, 1983).

[14]In a richer setting with limited commitment and imperfect record keeping (Kocherlakota (1998)), credit is constrained, so trades must engage in *quid pro quo*, involving a transaction medium (Kiyotaki and Wright (1989)). Cavalcanti and Wallace (1999) show that bankers arise as issuers of inside money when their trading history is public knowledge. Ostroy and Starr (1990) and Williamson and Wright (2010) review the literature of monetary theories.

immune to the asymmetric information problem in secondary markets.

This paper takes the aforementioned literature as a starting point: firms are assumed to hold liquidity in the form of safe debt issued by banks ("deposits").[15] Let m_0 denote a firm's deposit holdings *per unit of capital*. Investment at $t = 1$ is thus directly tied to deposits carried from $t = 0$:

$$i_1 k_0 \leq m_0 k_0.^{16} \tag{4}$$

Equation (4) resembles a money-in-advance constraint (e.g., Svensson (1985); Lucas and Stokey (1987)), except that what firms hold for transaction purposes is not fiat money, but bank debt, or "inside money." Thus, banks add value to the economy by supplying deposits that can be held by firms to relax this "money-in-advance" constraint on investment. Linking firm cash holdings to bank debt is in line with evidence.[17] Pozsar (2014) shows that corporate treasury, as one of the major cash pools, feeds leverage to the financial sector in the run-up to the global financial crisis.

Liquidity creation capacity. To impose more structure on the analysis, I assume loans take a particular contractual form: each dollar of loan extended at $t = 0$ is backed by a designated capital as collateral, and is repaid with interest rate R_0 at the end of date 1 if the collateral is intact. Thus, a fraction $\pi(Z_1)$ of loans default as their collateral is destroyed. The return to a diversified loan portfolio is $[1 - \pi(Z_1)](1 + R_0)$. To mimic the corresponding continuous-time expressions, I approximate it with $1 + R_0 - \pi(Z_1)$, ignoring $\pi(Z_1) R_0$, product of two percentages. This setup is similar to Klimenko et al. (2016).

Because of the aggregate shock, banks' safe debt capacity depends on their equity cushion. Let r_0 denote the deposit rate, and x_0 denote the leverage (asset-to-equity ratio). A banker will never default if her net worth is still positive even in a bad state ($Z_1 = -1$), i.e.,

$$\underbrace{x_0 e_0}_{total\ assets} \underbrace{[1 + R_0 - \pi_D(-1)]}_{loan\ return} \geq \underbrace{(x_0 - 1)e_0}_{total\ debt} \underbrace{(1 + r_0)}_{deposit\ repayment} .$$

This incentive or solvency constraint can be rewritten as a limit on leverage:

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[15]The concavity of investment technology $F(\cdot)$ also implies that firms prefer safe assets as liquidity buffer.

[16]To be consistent with the continuous-time expressions, deposits' interest payments are ignored in Equation (4).

[17]Based on Financial Accounts of the United States, Figure 1 in Online Appendix shows 80% of liquidity holdings of nonfinancial corporate businesses are in financial intermediaries' debt, with the rest dominated by Treasury securities.

$$x_0 \leq \frac{r_0 + 1}{r_0 + \delta + \sigma - R_0} := \overline{x}_0 \qquad (5)$$

Finally, I introduce the third and last friction – banks' equity issuance cost. At $t = 0$, bankers may raise equity subject to a proportional dilution cost χ. To raise one dollar, a bank needs to give $1 + \chi$ worth of equity to investors.[18] I will consider $\chi < \infty$ in the continuous-time analysis. For now, $\chi = \infty$ and banks do not issue equity. As a result, liquidity creation is limited by bankers' equity or balance-sheet capacity: total deposits cannot exceed $(\overline{x}_0 - 1) E_0$.

The frictions form three pillars of the model: firms' liquidity demand, bank debt as liquidity, and banks' equity constraint. Insufficient liquidity supply leads to underinvestment by compromising entrepreneurs' liquidity management. The next section recasts the model in a continuous-time framework and delivers the main results. Before that, I will close this section by showing several features of the static model that are shared with the continuous-time Markov equilibrium.

2.2 Equilibrium

Lemma 1, 2, and 3 below summarize the optimal choices for firms and banks at $t = 0$. We will focus on an equilibrium where firms' liquidity constraint binds (i.e., $i_1 = m_0$). One more unit of deposits can be used to purchase one more unit of goods as inputs to create $F'(m_0)$ more units of capital (with productivity α) when λ shock hits. This has an expected net value of $\lambda [\alpha F'(m_0) - 1]$, making firms willing to accept a return lower than ρ, the discount rate.[19] The spread, $\rho - r_0$, is *liquidity premium*, a carrying cost. Note that when $r_0 < \rho$, households do not hold deposits.

Lemma 1 (Liquidity Demand) *Firms' equilibrium deposits, m_0, satisfy the condition*

$$\lambda [\alpha F'(m_0) - 1] = \rho - r_0.$$

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[18]χ is a reduced form representation of informational frictions in settings such as Myers and Majluf (1984) or Dittmar and Thakor (2007).

[19]Because bankers' only endowments are goods that cannot be stored, to carry net worth to date 1, bankers must lend some goods to firms in exchange for loans, i.e., the instruments that bankers use to transfer wealth intertemporarily. Since goods cannot be stored, entrepreneurs must consume at $t = 0$ in equilibrium. To make risk-neutral entrepreneurs indifferent between consumption and savings, the price of capital q_0^K adjusts so that acquiring capital delivers an expected return equal to ρ, which is precisely the opportunity cost of holding deposits instead of capital.

Firms also choose the amount they borrow from banks, which is subject to the collateral constraint that the expected repayment cannot exceed the total pledgeable value ($\alpha \left(1 - \delta\right) k_0$). Given the expected default probability $\mathbb{E}\left[\pi\left(Z_1\right)\right] = \delta$, the expected loan repayment is $\left(1 - \delta\right)\left(1 + R_0\right)$ per dollar borrowed, approximated by $1 + R_0 - \delta$ (the product of two percentages ignored). When $R_0 - \delta = \rho$, firms are indifferent; when $R_0 - \delta < \rho$, firms borrow to the maximum. The spread, $\rho - \left(R_0 - \delta\right)$, is collateral shadow value κ_0 (the Lagrange multiplier of collateral constraint).

Lemma 2 (Credit Demand) *The equilibrium loan rate is given by:* $R_0 = \delta + \rho - \kappa_0$.

Competitive bankers take as given the market loan rate R_0 and deposit rate r_0. At $t = 0$, a representative banker chooses consumption-to-equity ratio y_0 (and retained equity $e_0 - y_0 e_0$), and the asset-to-equity ratio x_0 (leverage). Each dollar of retained equity is worth $x_0 \left[1 + R_0 - \pi\left(Z_1\right)\right] - \left(x_0 - 1\right)\left(1 + r_0\right)$ at $t = 1$, which is the difference between asset and liability value. Because $\mathbb{E}\left[\pi_D\left(Z_1\right)\right] = \delta$, the expected return on retained equity is $1 + r_0 + x_0 \left(R_0 - \delta - r_0\right)$. The return in a bad state is $1 + r_0 + x_0 \left(R_0 - \delta - \sigma - r_0\right)$. Let ξ_0 denote the Lagrange multiplier of the solvency constraint, i.e., the shadow value of bank equity.[20] The value function is

$$v\left(e_0; R_0, r_0\right) = \max_{y_0 \geq 0, x_0 \geq 0} y_0 e_0 + \frac{\left(e_0 - y_0 e_0\right)}{\left(1 + \rho\right)} \{1 + r_0 + x_0 \left(R_0 - \delta - r_0\right)$$
$$+ \xi_0 \left[1 + r_0 + x_0 \left(R_0 - \delta - \sigma - r_0\right)\right]\}.$$

Lemma 3 (Bank Optimization) *The first-order condition (F.O.C.) for bank leverage x_0 is*

$$R_0 - r_0 = \delta + \gamma_0^B \sigma, \tag{6}$$

where $\gamma_0^B = \left(\frac{\xi_0}{1 + \xi_0}\right) = \frac{R_0 - \delta - r_0}{\sigma} \in [0, 1)$ *is the banker's effective risk aversion or price of risk. Substituting the F.O.C. into the value function, we have*

$$v(e_0; q_0^B) = y_0 e_0 + q_0^B \left(e_0 - y_0 e_0\right), \text{ where, } q_0^B = \frac{\left(1 + r_0\right)\left(1 + \xi_0\right)}{\left(1 + \rho\right)}. \tag{7}$$

The banker consumes if $q_0^B \leq 1$; *if* $q_0^B > 1$, $y_0 = 0$ *so that the entire endowments are lent out.*

The equilibrium credit spread, $R_0 - r_0$, has two components: the expected default probability δ and the risk premium $\gamma_0^B \sigma$. Each dollar lent adds σ units of downside risk at date 1, which tightens

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[20]Note that ξ_0 is known at $t = 0$, so its subscript is 0 instead of 1.

the capital adequacy constraint. γ_0^B is the price of risk charged by bankers, the Sharpe ratio of risky lending financed by risk-free deposits. q_0^B is the marginal value of bank equity (Tobin's Q). Retained equity has a compounded payoff of $(1 + r_0)(1 + \xi_0)$ from reducing the external financing (debt) cost and relaxing the solvency constraint, so its present value is $\frac{(1+r_0)(1+\xi_0)}{(1+\rho)}$. When $q_0^B > 1$, bankers lend out all endowments in order to carry their wealth to $t = 1$ in the form of loans.

Substituting the equilibrium loan rate into Equation (6), we can solve for the liquidity premium $\rho - r_0$, as the sum of $\gamma_0^B \sigma$, bankers' risk compensation, and κ_0, the collateral shadow value.

Proposition 1 (Liquidity Premium Decomposition) *The equilibrium liquidity premium is given by*

$$\rho - r_0 = \gamma_0^B \sigma + \kappa_0. \tag{8}$$

Equation (8) decomposes the liquidity premium into an *intermediary wedge*, $\gamma_0^B \sigma$, that measures the scarcity of bank equity, and a collateral wedge κ_0. Since the liquidity premium equals the expected value of foregone marginal investment (Lemma 1), Equation (8) offers an anatomy of investment inefficiency. To support the first-best investment, i_{FB}, each firms must carry at least $i_{FB} K_0$ deposits in aggregate, which requires a minimum level of bank equity:

Condition 1 $E_0 \geq E_{FB}$, where $E_{FB} := \frac{i_{FB} K_0}{\overline{x}_{FB} - 1} = \frac{i_{FB}}{\frac{1+\rho}{\sigma} - 1} K_0$, and i_{FB} is defined in Equation (1).

$\overline{x}_{FB}$ is solved as follows: under the first-best investment, the liquidity premium is zero, so $\kappa_0 = 0$. Substituting $r_0 = \rho$ and $R_0 = \delta + \rho$ into the solvency constraint yields $\overline{x}_{FB} = \frac{1+\rho}{\sigma}$. When the size of aggregate shock is larger (higher σ), the required bank equity as risk buffer (E_{FB}) is larger.

First-best deposit creation also requires a minimum stock of collateral to back bank loans. The minimum bank lending that supports the first-best investment is $\overline{x}_{FB} E_{FB}$, so that collateral must be sufficient to cover firms' expected debt repayment: $\alpha (1 - \delta) K_0 \geq \overline{x}_{FB} E_{FB} (1 + \rho)$, or,

$$K_0 \geq K_{FB} := \frac{\overline{x}_{FB} E_{FB} (1 + \rho)}{\alpha (1 - \delta)} = \left(\frac{\frac{1+\rho}{\sigma}}{\frac{1+\rho}{\sigma} - 1} \right) \left(\frac{i_{FB} (1 + \rho)}{\alpha (1 - \delta)} \right) K_0,$$

This condition can be simplified into the following parameter restrictions:

Condition 2 $\frac{\alpha(1-\delta)}{1+\rho} \geq \left(\frac{1}{1 - \frac{\sigma}{1+\rho}} \right) i_{FB}$, where i_{FB} is defined in Equation (1).

15

Condition 2 is more likely to be violated when the expected collateral destruction rate δ is higher. Thus, δ measures the severity of collateral shortage that is studied by Holmström and Tirole (1998).[21] This paper focuses on the scarcity of intermediation capacity. As shown in Condition 1, such scarcity is more severe if σ is larger. Therefore, two parameters, δ and σ, correspond to the strengths of two limits on liquidity creation. Corollary 1 summarizes the analysis.

Corollary 1 (Sufficient Conditions for Liquidity Shortage) *The equilibrium liquidity premium is positive, and investment is below the first-best level, if either Condition 1 or 2 is violated.*

3 Dynamic Model: Procyclical Liquidity Creation

To study the cyclicality of bank leverage and the frequency and duration of crisis, I recast the model in continuous time. New mechanisms arise from agents' intertemporal decision making. The analysis focuses on the intermediary wedge, assuming a corresponding version of Condition 2 holds, so the economy has enough capital as collateral, but not enough bank equity as risk buffer.

3.1 Setup

Continuous-time setup. All agents maximize risk-neutral life-time utility with discount rate ρ. Households consumes the generic goods and can invest in securities issued by firms and banks.[22] Firms trade capital at price q_t^K. One unit of capital produces α units of goods per unit of time. They can issue equity to households, promising an expected return of ρ per unit of time (i.e., their cost of equity). Given the deposit rate r_t, the deposit carry cost or the liquidity premium, is defined by the spread between ρ and deposit rate r_t as in the static model.[23]

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[21]See also the literature of asset shortage, such as Woodford (1990b), Kocherlakota (2009), Kiyotaki and Moore (2005), Caballero and Krishnamurthy (2006), Farhi and Tirole (2012), Giglio and Severo (2012) among others.

[22]Risk-neutral households' required return is fixed at ρ because negative consumption is allowed, which is interpreted as dis-utility from additional labor to produce extra goods as in Brunnermeier and Sannikov (2014). Allowing negative consumption serves the same purpose as assuming large endowments of goods per period in the static model.

[23]Nagel (2016) emphasizes the variation in illiquid return (i.e. ρ in the model) as a driver of the liquidity premium dynamics in data. This paper provides an alternative model that focuses on the yield on money-like securities, r_t.

At idiosyncratic Poisson times (intensity λ), firms are hit by a liquidity shock and cut off from external financing. These firms either quit or invest. Let k_t denote a firm's capital holdings. Investing $i_t k_t$ units of goods can preserve the existing capital and create $F(i_t)k_t$ units of new capital. Investment is constrained by the firm's deposit holdings, $i_t \leq m_t$, where m_t is the deposits per unit of capital on its balance sheet.[24] Holding deposits allows firms' wealth to jump up at these Poisson times through the creation of new capital. I assume the technology $F(\cdot)$ is sufficiently productive, so we focus on an equilibrium where the liquidity constraint always binds.

The aggregate shock Z_t is a standard Brownian motion. Every instant, $\delta dt - \sigma dZ_t$ fraction of capital is destroyed. Firms default on loans backed by the destroyed capital. Let R_t denote the loan rate. For one dollar borrowed from banks at t, firms expect to pay back

$$\underbrace{(1 + R_t dt)}_{\text{principal + interest payments}} \quad [1 - \underbrace{(\delta dt - \sigma dZ_t)}_{\text{default probability}}] = 1 + R_t dt - (\delta dt - \sigma dZ_t),$$

where high-order infinitesimal terms are ignored. The default probability is a random variable that loads on dZ_t.[25] Both loans and deposits are short-term contracts, initiated at t and settled at $t+dt$.[26]

Let r_t denote the deposit rate, and x_t banks' asset-to-equity ratio. Let c_t^B denote a bank's cumulative dividend. $dc_t^B > 0$ means consumption and paying dividends to outside shareholders (households); $dc_t^B < 0$ means raising equity. As in the static model, we can define $dy_t = \frac{dc_t^B}{e_t}$ as the payout or issuance ratio, which is an impulse variable, so bank equity e_t follows a regulated diffusion process, reflected at payout and issuance (i.e., when $dy_t \neq 0$):

$$de_t = \underbrace{e_t x_t}_{\text{Loan value}} \underbrace{[R_t dt - (\delta dt - \sigma dZ_t)]}_{\text{Loan return}} - \underbrace{e_t (x_t - 1)}_{\text{Debt value}} \underbrace{r_t dt}_{\text{Interest payment}} - \underbrace{e_t dy_t}_{\text{Payout or issue}} - \underbrace{e_t \iota dt}_{\text{Cost of operations}} . \quad (9)$$

Because in equilibrium, banks earn a positive expected return on equity, the operation cost ι is

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[24]The idiosyncratic nature of liquidity shock and the assumption that firms can access external funds in normal times imply that firms' liquidity demand does not contain hedging motive that complicates model mechanism. Bolton, Chen, and Wang (2013) model the market timing motive of corporate liquidity holdings in the presence of technological illiquidity and state-dependent external financing costs. He and Kondor (2016) examine how the hedging motive of liquidity holdings amplifies investment cycle through pecuniary externality in the market of productive capital.

[25]Probit transformation can guarantee $\pi(dZ_t) \in (0, 1)$ but complicates expressions. See also Klimenko et al. (2016).

[26]I assume banks repay deposits after investment takes place, so that investing firms cannot be buy-and-hold investors, and thus, have to actually sell deposits in exchange for goods. Long-term deposits avoid this assumption, but would introduce other mechanisms, such as the Fisherian deflationary spiral in Brunnermeier and Sannikov (2016). Similarly, long-term loan contracts introduce the fire sale mechanism in Brunnermeier and Sannikov (2014).

introduced to motivate payout so that the banking sector does not outgrow the economy.[27]

Bankers maximize life-time utility, subject to a proportional equity issuance cost:

$$\mathbb{E}\left\{\int_{t=0}^{\tau} e^{-\rho t}\left[\mathbb{I}_{\{dy_t \geq 0\}} - (1+\chi)\,\mathbb{I}_{\{dy_t < 0\}}\right] e_t dy_t\right\}.\tag{10}$$

$\mathbb{I}_A$ is the indicator function of event A.[28] The solvency constraint in the static setting boils down to the requirement of non-negative equity. Unlike the static setting, in equilibrium, bankers always preserve a slackness, so $\tau := \inf\{t : e_t \leq 0\} = \infty$. As will be shown later, even in the absence of a binding solvency constraint, bankers are still risk-averse due to the recapitalization friction χ.

State variable. At time t, the economy has K_t units of capital and aggregate bank equity E_t. In principle, a time-homogeneous Markov equilibrium would have both as state variables. Because production has constant return-to-scale and the investment technology is homogeneous of degree one in capital, the Markov equilibrium has only one state variable:

$$\eta_t = \frac{E_t}{K_t}.$$

Since the model highlights the interaction between liquidity supply and demand, intuitively, η_t measures the size of liquidity suppliers (banks) relative to that of liquidity demanders (firms).

Because there is a unit mass of homogeneous bankers, E_t follows the same dynamics as e_t, so the instantaneous expectation and standard deviation of $\frac{dE_t}{E_t}$ (μ_t^e and σ_t^e) are $r_t + x_t(R_t - \delta - r_t) - \iota$ and $x_t\sigma$ respectively (from Equation (9)). By Itô's lemma, η_t follows a regulated diffusion process

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dZ_t - dy_t,\tag{11}$$

where μ_t^η is $\mu_t^e - [\lambda F(m_t) - \delta] - \sigma_t^e\sigma + \sigma^2$, with bracket term being the expected growth rate of K_t, and the shock elasticity σ_t^η is $(x_t - 1)\sigma$, which is positive because banks lever up ($x_t > 1$). Positive shocks increase η_t, so banks become relatively richer; negative shocks make banks relatively undercapitalized. As η_t evolves over time, the economy repeats the timeline in Figure 1

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[27]The cost of operations is mathematically equivalent to a higher time-discount rate for bankers, common in the literature of heterogeneous-agent models (e.g., Kiyotaki and Moore (1997)). It can also be interpreted as agency cost.

[28]In different settings, Van den Heuvel (2002), Phelan (2016), and Klimenko et al. (2016) also introduce issuance frictions in dynamic banking models. Dilution cost is just one form of frictions that lead to the endogenous variation of intermediaries' risk-taking capacity. He and Krishnamurthy (2012) use a minimum requirement of insiders' stake.

with date 0 replaced by t and date 1 replaced by $t + dt$. Let intervals $\mathbb{B} = [0, 1]$ and $\mathbb{K} = [0, 1]$ denote the sets of banks and firms respectively. The Markov equilibrium is formally defined below.

Definition 1 (Markov Equilibrium) *For any initial endowments of firms' capital $\{k_0(s), s \in \mathbb{K}\}$ and banks' goods (i.e., initial bank equity) $\{e_0(s), s \in \mathbb{B}\}$ such that*

$$\int_{s \in \mathbb{K}} k_0(s)ds = K_0, \text{ and } \int_{s \in \mathbb{B}} e_0(s)ds = E_0,$$

a Markov equilibrium is described by the stochastic processes of agents' choices and price variables on the filtered probability space generated by the Brownian motion $\{Z_t, t \geq 0\}$, such that:

(i) Agents know and take as given the processes of price variables, such as the price of capital, the loan rate, and the deposit rate (i.e., agents are competitive with rational expectation);

(ii) Households optimally choose consumption and savings that are invested in securities issued by firms and banks;

(iii) Firms optimally choose capital holdings, deposit holdings, investment, and loans;

(iv) Bankers optimally choose leverage, and consumption/payout and issuance policies;

(v) Price variables adjust to clear all markets with goods being the numeraire;

(vi) All the choice variables and price variables are functions of η_t, so Equation (11) is an autonomous law of motion that maps any path of shocks $\{Z_s, s \leq t\}$ to the current state η_t.

3.2 Markov Equilibrium

The risk cost of liquidity creation. In analogy to Proposition 1, I will show a risk cost of liquidity creation that ties liquidity supply to bank equity. I start with firms' demand for bank deposits.

Lemma $1'$ gives firms' optimal deposit demand in analogy to Lemma 1, with one modification that capital is valued at the market price q_t^K instead of the terminal value α in the static setting.[29] As will be shown, this difference is critical, as it leads to a unique intertemporal feed-

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[29]To be precise, the liquidity shock hits at $t + dt$, and by then the capital created will be worth $q_{t+dt}^K = q_t^K + dq_t^K$. In equilibrium, q_t^K is a diffusion process with continuous sample paths, so dq_t^K is infinitesimal, and thus, ignored.

back mechanism that amplifies the procyclicality of liquidity creation. As in the static model, households do not hold deposits when $r_t < \rho$, so firms' deposit demand is the aggregate demand.

Lemma 1′ (Liquidity Demand) *Firms' equilibrium deposits, m_t, satisfy the condition*

$$\lambda \left[q_t^K F'(m_t) - 1 \right] = \rho - r_t. \tag{12}$$

The static model highlights two limits on liquidity creation: collateral scarcity and bank equity. I will focus on the latter, and then confirm that firms' collateral constraint never binds in the calibrated equilibrium. Moreover, shutting down the collateral channel also isolates the mechanism in this paper from mechanisms that emphasize the role of endogenous asset/collateral value via binding collateral constraints (e.g., Kiyotaki and Moore (1997)). Under this assumption, the expected loan repayment $R_t - \delta$, is equal to ρ, i.e., the collateral wedge drops out.

Lemma 2′ (Credit Demand) *The equilibrium loan rate is given by: $R_t = \delta + \rho$.*

Bankers solve a dynamic problem. Following Lemma 3, I conjecture bankers' value function is linear in equity, $v\left(e_t; q_t^B\right) = q_t^B e_t$, where q_t^B summarizes the investment opportunity set. Define ϵ_t^B as the elasticity of q_t^B: $\epsilon_t^B := \frac{dq_t^B / q_t^B}{d\eta_t / \eta_t}$. Intuitively, q_t^B signals the scarcity of bank equity, so I look for an equilibrium in which $\epsilon_t^B \leq 0$. Individual bankers take as given the equilibrium dynamics of q_t^B. Let μ_t^B and σ_t^B denote the instantaneous expectation and standard deviation of $\frac{dq_t^B}{q_t^B}$ respectively. The Hamilton-Jacobi-Bellman (HJB) equation can be written as

$$\rho = \max_{dy_t^B \in \mathbb{R}} \left\{ \frac{\left(1 - q_t^B\right)}{q_t^B} \mathbb{I}_{\{dy_t > 0\}} dy_t + \frac{\left(q_t^B - 1 - \chi\right)}{q_t^B} \mathbb{I}_{\{dy_t < 0\}} \left(-dy_t\right) \right\} \tag{13}$$
$$+ \mu_t^B + \max_{x_t \geq 0} \left\{ r_t + x_t \left(R_t - \delta - r_t\right) - x_t \gamma_t^B \sigma \right\} - \iota,$$

where the effective risk aversion is defined by $\gamma_t^B := -\sigma_t^B$. By Itô's lemma, $\gamma_t^B = -\epsilon_t^B \sigma_t^\eta \geq 0$.

Lemma 3′ (Bank Optimization) *The first-order condition for bank leverage x_t is*

$$R_t - \delta - r_t = \gamma_t^B \sigma, \tag{14}$$

The banker pays dividends ($dy_t > 0$) if $q_t^B \leq 1$, and raises equity ($dy_t < 0$) if $q_t^B \geq 1 + \chi$.

Bankers' issuance and payout policies imply that η_t is bounded by two reflecting boundaries: the issuance boundary $\underline{\eta}$, given by $q^B\left(\underline{\eta}\right) = 1+\chi$, and the payout boundary $\overline{\eta}$ given by $q^B\left(\overline{\eta}\right) = 1$. When η_t falls to $\underline{\eta}$, banks raise equity and η_t never decreases further; When η_t rises to $\overline{\eta}$, banks pay out dividends and η_t never increases further. When $\eta_t \in \left(\underline{\eta}, \overline{\eta}\right)$, bankers neither issue equity nor pay out dividends, because $q_t^B \in (1, 1+\chi)$ by the monotonicity of $q^B(\eta_t)$. As η_t rises following good shocks and falls following bad shocks, banks follow a countercyclical equity management strategy, paying out dividends in good times and issuing shares in bad times, which is consistent with the evidence (Adrian, Boyarchenko, and Shin, 2015).

Lemma 4 (Reflecting Boundaries) *The economy moves within bank issuance boundary $\underline{\eta}$ and payout boundary $\overline{\eta}$. In $\left[\underline{\eta}, \overline{\eta}\right]$, the law of motion of the state variable η_t is given by Equation (11).*

Bankers are risk-averse because of the recapitalization friction. From an individual banker's perspective, the issuance cost causes her marginal value of equity q_t^B to be negatively correlated with shocks. Following a negative shock, bankers will not raise equity unless q_t^B reaches $1 + \chi$, so the whole industry shrinks (i.e. the *aggregate* bank equity decreases), and intuitively, Tobin's Q, q_t^B, increases. Following a positive shock, bankers will not immediately pay out dividends unless q_t^B drops to 1, so the whole industry expands and q_t^B decreases. Thus, bankers require a risk premium for holding any asset whose return is positively correlated with dZ_t (i.e., negatively correlated with q_t^B). In particular, bankers require a risk compensation for extending loans.

On the left-hand side of Equation (14) is the net interest margin, $R_t - \delta - r_t$, the marginal benefit of issuing deposits backed by risky loans. The right-hand side is the marginal cost, that is the σ units of risk exposure, priced at γ_t^B per unit.[30] We can interpret the equilibrium γ_t^B as the expected profit per unit of risk (i.e. the Sharpe ratio), from creating deposits backed by risky loans:

$$\gamma_t^B = \frac{R_t - \delta - r_t}{\sigma}.$$

Banks face two markets, the loan market and the money market. With the loan rate R_t equal to $\rho + \delta$ (Lemma 2'), there is a one-to-one mapping between the deposit rate r_t and γ_t^B.

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[30]$\gamma_t^B \sigma$ opens up a wedge between the credit spread, $R_t - r_t$, and δ the expected default rate. This intermediary premium shares the insight of He and Krishnamurthy (2013), but here, the purpose of intermediation is to create liquidity. Bankers need loans to back deposits, and all that households need is to break even as shown in Lemma 2'.
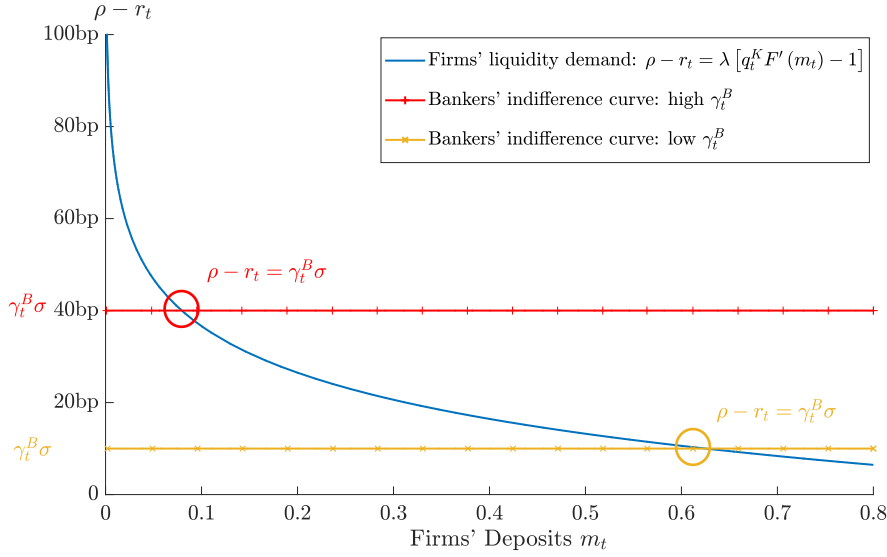
Figure 2: **Deposit Market.** This figure plots the deposit demand curve of firms and two indifference curve of banks corresponding to high and low γ_t^B respectively. The intersection points are the deposit market equilibrium.

Interpreting γ_t^B as the Sharpe ratio or profitability of liquidity creation helps us build an intuitive connection between q_t^B and γ_t^B. As a summary statistic for banks' investment opportunity set, q_t^B reflects the expectation of future profits from liquidity creation (i.e. the future paths of γ_t^B). Intuitively, when the banking sector is relatively large, i.e., η_t is high, banks' profit per unit of risk, γ_t^B, declines. This is a key equilibrium property, later confirmed by the full solution. Substituting the equilibrium loan rate into Equation (14), we have the dynamic counterpart of Proposition 1.[31]

Proposition 1′ (Liquidity Premium) *The equilibrium liquidity premium is given by*

$$\rho - r_t = \gamma_t^B \sigma. \tag{15}$$

Figure 2 takes a snapshot of the deposit market, given capital value q_t^K and γ_t^B. In the Markov equilibrium, these variables vary continuously with η_t. The horizontal axis is m_t, the representative firm's deposits per unit of capital. The vertical axis is the liquidity premium. The investment technology $F(\cdot)$ is concave, so firms' indifference curve from Lemma 1′ gives a downward-sloping demand curve. The supply curve is represented by bankers' indifference curve $\rho - r_t = \gamma_t^B \sigma$. Two

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[31]Recall that for the transparency of the dynamic mechanisms, I assume firms' collateral constraint never binds, so the collateral shadow value disappears. This assumption is confirmed later by the solution of calibrated model.

equilibrium points are circled. When banks undercapitalized (low η_t), γ_t^B is high. The equilibrium liquidity premium must compensate bankers' risk exposure from issuing deposits backed by risky loans. When banks are well capitalized (high η_t), γ_t^B is low, and the liquidity premium is low.[32]

With deposits being risk-free and loan risky, liquidity creation induces risk mismatch on bank balance sheets, with precisely σ units of risk per dollar of liquidity created. As γ_t^B varies with η_t, this *risk cost* of liquidity production links banks' balance-sheet capacity to the real economy through the liquidity constraint on firms' investments. This risk cost of liquidity creation adds to the literature of balance-sheet channel (Bernanke and Gertler (1989); Kiyotaki and Moore (1997)). By modeling banks as liquidty suppliers, this paper offers a bank balance-sheet perspective on liquidity shortage previously studied by Woodford (1990b) and Holmström and Tirole (1998).

Corollary 1′ (Investment Inefficiency) *From Lemma 1′ and Proposition 1′, we have*

$$\lambda \left[q_t^K F'(m_t) - 1 \right] = \gamma_t^B \sigma. \tag{16}$$

The risk compensation charged by bankers is exactly the net present value of foregone marginal investment. When banks are undercapitalized and γ_t^B is high, firms hold less liquidity and invest less. This result echoes Corollary 1 of the static model, except that now bank equity evolves endogenously. Bad shocks destroy bank equity, so liquidity supply contracts, slowing down resources reallocation towards investing firms. Following good shocks, more liquidity is created, facilitating reallocation. Eisfeldt and Rampini (2006) find procyclical reallocation among firms. Bachmann and Bayer (2014) find procyclical dispersion of firms' investment rates. Here, procyclical reallocation is driven by procyclical liquidity creation and investment.

So far, we have revisited the main results of the static model in a dynamic setting. Next, I will discuss intertemporal feedback mechanisms that amplify the procyclicality of liquidity creation.

Intertemporal feedback. The endogenous capital value plays a critical role in generating a feedback mechanism. Proposition 2 shows firms' indifference condition as a capital pricing formula.

Proposition 2 (Capital Valuation) *The equilibrium price of capital satisfies*

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[32]Drechsler, Savov, and Schnabl (2016) provide evidence on banks' unique position in creating deposits. In contrast with this paper, they emphasize banks' market power as the driver of deposit rate instead of balance-sheet capacity.
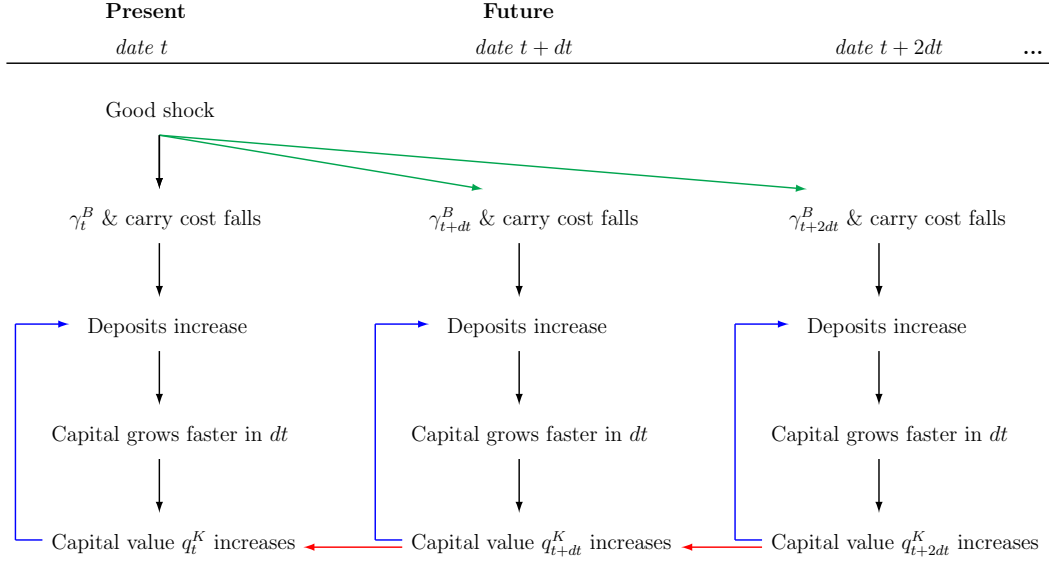
Present

date t

Future

date t + dt

date t + 2dt   ...

Good shock

γ_t^B & carry cost falls

γ_{t+dt}^B & carry cost falls

γ_{t+2dt}^B & carry cost falls

Deposits increase

Deposits increase

Deposits increase

Capital grows faster in dt

Capital grows faster in dt

Capital grows faster in dt

Capital value q_t^K increases

Capital value q_{t+dt}^K increases

Capital value q_{t+2dt}^K increases

Figure 3: **Intertemporal Feedback and Procyclicality.** This figure illustrates the mechanism behind the procyclicality of q_t^K. Following good shocks, bank risk price γ_t^B declines, and due to the persistence of shock impact, the path of expected γ_t^B shifts down. Firms face a lower liquidity premium now and hold more liquidity, and they expect so in the future, which translates into a higher growth path of capital in expectation and higher capital value.

$$q_t^K = \frac{\overbrace{\alpha}^{Production} + \overbrace{\lambda\left[q_t^K F(m_t) - m_t\right]}^{Expected\ net\ investment\ gain} - \overbrace{(\rho - r_t)m_t}^{Deposit\ carry\ cost}}{\underbrace{\rho}_{Discount\ rate} - (\underbrace{\mu_t^K}_{Expected\ price\ appreciation} - \underbrace{\delta}_{Expected\ capital\ destruction} + \underbrace{\sigma_t^K \sigma}_{Quadratic\ covariation})}, \quad (17)$$

where μ_t^K and σ_t^K are defined in the equilibrium price dynamics: $dq_t^K = \mu_t^K q_t^K dt + \sigma_t^K q_t^K dZ_t$.

Capital value q_t^K is procyclical. Consider a positive shock, $dZ_t > 0$, at an interior state, $\eta_t \in (\underline{\eta}, \overline{\eta})$. Since fewer loans default than expected, banks receive a windfall. Given the wedge between q_t^B and 1 that is created by the issuance cost, q_t^B does not immediately jump down to one and trigger payout. So, banks' equity increases, and *in expectation*, the shock's impact on the bank equity will only dissipate gradually into the future. Thus, a positive shock increases current bank equity, and due to the persistence of its impact, it lifts up the expectation of future bank equity.

Thus, a positive shock increases capital value through two channels. As banks' equity increases, they charge a lower price of risk for deposit creation, so firms pay a lower deposit carry cost, $\rho - r_t$, and hold more deposits from t to $t + dt$. Capital is expected to grow faster in dt thanks
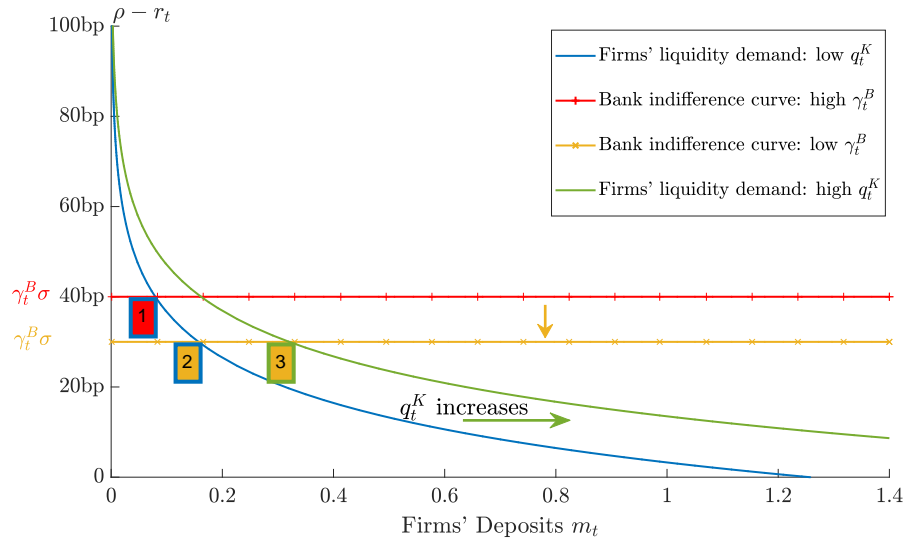
Figure 4: **Deposit Market Response to A Positive Shock.** First, the bank indifference curve shifts downward because bank risk price γ_t^B declines (i.e., from (1) to (2)). Then firms' deposit demand curve shifts outward because capital value q_t^K rises (i.e., from (2) to (3)), which is in turn due to a higher growth path in expectation as in Figure 3.

to more investments financed by these deposits, which directly leads to a higher market price of capital. This is the *contemporaneous* channel of procyclicality of q_t^K.

An *intertemporal* channel further increases capital value. Due to the persistent impact of the shock, firms expect the banking sector to be better capitalized for an extended period of time, and thereby, they expect to hold more deposits and capital to grow faster going forward. This lifts up the expectation of future capital value, which feeds back into an even higher current price through the expected price appreciation μ_t^K. Figure 3 illustrates the two channels of procyclical q_t^K.[33]

Procyclical bank leverage. Following good shocks, banks' equity increases and they charge a lower price of risk γ_t^B. As illustrated by Figure 4, the money market moves from "1" to "2'.'' The equilibrium quantity of deposits increases. Whether it increases faster or slower than banks' equity determines whether bank leverage is procyclical or countercyclical.

Because capital value q_t^K is procyclical, firms' liquidity demand will also shift outward, so the equilibrium point moves further from "2" to "3," which increases the equilibrium quantity of

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[33]Note that an additional channel of procyclical q_t^K has been shut down by the assumption that the economy has enough collateral to back loans. Following good shocks, banks expand and become willing to lend more. When firms' borrowing constraint binds, a collateral shadow value (κ in the static model) arises, which increases capital value.

deposits even further. This endogenous expansion of firms' liquidity demand allows banks' debt to grow faster than their equity, contributing to the procyclicality of bank leverage.

One aspect of the model that distinguishes itself from other macro-finance models is this endogenous expansion of the demand for intermediaries' debt. A static demand may lead to countercyclical leverage (e.g. He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014)).[34]

This paper shares with Kiyotaki and Moore (1997) the idea that intertemporal complementarity amplifies fluctuations. Capital becomes more valuable (higher q_t^K) because it grows faster, which is in turn due to more liquidity held by firms in the *future*. Through q_t^K, firms' *current* liquidity demand rises in the expectation of future market market conditions. The procyclicality of liquidity demand contributes to the procyclicality of bank leverage and the resulting risk accumulation. As leverage rises, bank equity becomes more sensitive to shocks, so does the whole economy through η_t. Asset price here plays a key role in intertemporal feedback, but it differs from a typical balance-sheet effect, for example in Kiyotaki and Moore (1997).

Even though the model features a specific type of procyclical liquidity demand motivated by corporate cash holdings, the insight that leverage procyclicality results from liquidity-demand procyclicality is general. We would expect a booming economy to have strong demand for liquid assets issued by intermediaries. The model assumes that when firms are not experiencing the Poisson liquidity shock, they can immediately respond to changes in capital value by raising funds from banks or households to build up savings. In reality, firms may not act so swiftly. To what extent it affects the procyclicality of bank leverage is an interesting empirical question.[35]

Dynamic investment inefficiency. The endogenous variation in capital value leads to a new form of investment inefficiency. Given q_t^K, Corollary 1′ reveals a form of *static inefficiency*, measured by the wedge between firms' deposits m_t and the contemporaneous investment target i_t^*, defined

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[34]In this type of models, intermediaries bet on asset prices. Asset price volatility is countercyclical, so a value-at-risk constraint can make leverage procyclical (Adrian and Boyarchenko (2012); Danielsson, Shin, and Zigrand (2012)).

[35]Related, since deposits can be regarded as insurance against the Poisson shock and the demand for such insurance is procyclical, this paper shares the insight of Rampini and Viswanathan (2010) that when firms are richer, they hedge more. In Rampini and Viswanathan (2010), hedging competes with investing for limited resources, and thus, richer firms, with more resources at hand, hedge more. In contrast, firms in this paper are always unconstrained outside of the Poisson times, so the procyclical demand for insurance is not driven by more resources available, but rather, by the procyclical benefit of hedging (i.e., more valuable investment), which is in turn due to the procyclicality of q_t^K.

by $q_t^K F'(i_t^*) = 1$. Static inefficiency is due to bankers' *current* capacity to create liquidity is limited. This echoes Corollary 1 of the static model. In a dynamic setting, the target i_t^* also varies with capital value. Due to the necessity and cost of carrying deposits, q_t^K is smaller than q_{FB}^K, the capital value in an unconstrained economy in which firms finance investment freely (defined below).

$$q_{FB}^K = \frac{\alpha + \lambda \left[q_{FB}^K F(i_{FB}) - i_{FB} \right]}{\rho + \delta}, \tag{18}$$

where the first-best investment is given by $q_{FB}^K F'(i_{FB}) = 1$. Because $q_t^K < q_{FB}^K$, the target i_t^* is below the first-best investment rate i_{FB}. Since q_t^K incorporates the expectation of future paths of deposit carry cost (liquidity premium), $i_{FB} - i_t^*$, measures a form of *dynamic inefficiency*.[36]

Stagnation and instability. Recession states are states with negative expected growth rate of K_t. Growth is driven by investment, which is tied to firms' liquidity holdings. Negative shocks deplete banks' equity and elevate the required risk compensation $\gamma_t^B \sigma$. At the same time, capital value decreases. As illustrated by Figure 4, bankers' indifference curve shifts upward and firms' liquidity demand curve shifts inward, so firms hold less deposits and invest less, and the economy grows slower. Banking crises affect the real economy through the contraction of liquidity supply, which echoes the classic account of the Great Depression by Friedman and Schwartz (1963).

Procyclical bank leverage implies stagnant recessions. Recessions happen near the issuance boundary $\underline{\eta}$ where banks are undercapitalized (η_t is low) and leverage is low. Low leverage limits the impact of good shocks on bank equity, so banks have to rebuild equity slowly. Low leverage also limits the impact of bad shocks, but this benefit is small. Near $\underline{\eta}$, the impact of bad shocks is already bounded: bank equity never decreases below $\underline{\eta}$. Low leverage and such *asymmetric impact* of shocks near boundary implies the economy is stuck with undercapitalized banks for a long time.

Procyclical leverage leads to downside risk accumulation in booms. Following good shocks, banks build up equity and leverage. As the economy approaches the payout boundary $\bar{\eta}$, shock impact becomes increasingly asymmetric. Since bank equity never rises above $\bar{\eta}$, the impact of good shocks is bounded, so high leverage only serves to amplify the impact of bad shocks on bank

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[36]Note that this dynamic inefficiency is about the lack of investment (productive reallocation across firms), *not* the dynamic inefficiency in overlapping-generation models that is due to the lack of intergenerational trade.

equity. Therefore, as a boom prolongs, leverage builds up, and the economy becomes increasingly fragile. Even small bad shocks can significantly deplete bank equity and trigger a recession.

Proposition 3 solves the stationary probability density of η_t (i.e., the likelihood of different states in the long run) and the expected time to reach $\eta \in [\underline{\eta}, \overline{\eta}]$ from $\underline{\eta}$ ("recovery time").

Proposition 3 *The stationary probability density of state variable η_t, $p(\eta)$ can be solved by:*

$$\mu^\eta (\eta) \, p(\eta) - \frac{1}{2} \frac{d}{d\eta} \left(\sigma^\eta (\eta)^2 \, p(\eta) \right) = 0,$$

where $\mu^\eta (\eta)$ and $\sigma^\eta (\eta)$ are defined in Equation (11). The expected time to reach η from $\underline{\eta}$, $g (\eta)$ can be solved by:

$$1 - g' (\eta) \, \mu^\eta (\eta) - \frac{\sigma^\eta (\eta)^2}{2} g'' (\eta) = 0,$$

with the boundary conditions $g \left(\underline{\eta} \right) = 0$ and $g' \left(\underline{\eta} \right) = 0$.

Solving the equilibrium. The solution is a set of functions defined on $[\underline{\eta}, \overline{\eta}]$. Each function maps the value of state variable η_t to the value of an endogenous variable. These functions are separated into two sets. The first set includes the forward-looking variables $\left(q^B (\eta_t), q^K (\eta_t) \right)$. The second includes variables, such as banks' leverage x_t, firms' deposits-to-capital ratio m_t, and deposit rate r_t that can be solved once we know the first set of functions. We solve the second set of variables as functions of $\left(q^B (\eta_t), q^K (\eta_t) \right)$ and their derivatives to transform Equation (13) and (17) into a system differential equations of $\left(q^B (\eta_t), q^K (\eta_t) \right)$ using Itô's lemma.

A key step is to solve bank leverage from the intersection of liquidity demand and supply curves. We can use the deposit market clearing condition

$$m_t K_t = (x_t - 1) \, E_t, \text{ i.e., } m_t = (x_t - 1) \, \eta_t,$$

to substitute out m_t with $(x_t - 1) \, \eta_t$ on the left hand side of Equation (16). On the right hand side is the intermediary wedge, $\gamma_t^B \sigma$. By knowing the function $q^B (\eta_t)$, we know the elasticity ϵ_t^B, so banks' risk price, γ_t^B, is directly linked to the equilibrium leverage as follows.

$$\gamma_t^B = -\epsilon_t^B \sigma_t^\eta, \text{ where the } \eta_t\text{'s instantaneous shock elasticity is } \sigma_t^\eta = (x_t - 1) \, \sigma. \tag{19}$$

Equation (16) has a unique solution ($F'' (\cdot) < 0$) of leverage x_t as a function of η_t, q_t^K, and ϵ_t^B. m_t is given by deposit market clearing condition, and r_t from Equation (12). Details are in Appendix

I, which also shows the existence and uniqueness of Markov equilibrium in a constructive manner.

Proposition 4 (Markov Equilibrium) *There exists a unique Markov equilibrium with state variable η_t that follows an autonomous law of motion in $\left[\underline{\eta}, \overline{\eta}\right]$. Given functions $q^B(\eta_t)$ and $q^K(\eta_t)$, agents' optimality conditions and market clearing conditions solve bank leverage, firms' deposits, and deposit rate as functions of η_t. Substituting these variables into bankers' HJB equation and the capital pricing formula (Equations (13) and (17)), we have a system of two second-order ordinary differential equations that solves $q^B(\eta_t)$ and $q^K(\eta_t)$ under the following boundary conditions:*

At $\underline{\eta}$: (1) $\frac{dq^K(\eta_t)}{d\eta_t} = 0$; (2) $q^B(\overline{\eta}) = 1 + \chi$; (3) $\frac{d\left(q_t^B \eta_t\right)}{d\eta_t} = 0$;

At $\overline{\eta}$: (4) $\frac{dq^K(\eta_t)}{d\eta_t} = 0$; (5) $q^B\left(\underline{\eta}\right) = 1$; (6) $\frac{d\left(q_t^B \eta_t\right)}{d\eta_t} = 1$.

We need exactly six boundary conditions for two second-order ordinary differential equations and two endogenous boundaries to pin down the solution. (1) and (4) prevent capital value from jumping upon reflection, ruling out arbitrage in the market of capital. (2) and (5) are the value-matching conditions for banks' issuance and payout respectively. (3) and (6) are the smooth-pasting conditions that guarantee that the bank shareholders' value does not jump at the reflecting boundaries (similar to those in Brunnermeier and Sannikov (2014) and Phelan (2016)).[37]

3.3 Solution

Calibration. To numerically solve the differential equations, I calibrate the model as follows. One unit of time is set to one year. δ and σ are the mean and standard deviation of loan delinquency rates (source: FRED). The other parameters are set to match model moments to data, such as interest rate, corporate cash holdings, and economic growth. All model moments are based on the stationary distribution. I use the mean and standard deviation of liquidity premium (GC repo/T-bill spread in Nagel (2016)) to calibrate λ, the arrival rate of liquidity shocks, and χ, the issuance cost that governs the tail behavior of the model. Since in reality, liquidity premium varies due to forces

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[37]The market value of bank equity is $q_t^B \eta_t K_t$. (3) guarantees the value of existing shares does not jump when banks issue news shares. (6) guarantees the value of bank equity declines exactly by the amount of dividends paid out. If (3) or (6) is violated, taking as given aggregate issuance and payout, individual banks have incentive to deviate.

beyond the model mechanism, χ is set conservatively so that the model generates half of the data standard deviation. The qualitative implications are robust to the choice of χ. Bank leverage is intentionally left out of the calibration, so the leverage dynamics, and the associated boom-bust cycle, may serve for external validation. Appendix II summarizes the calibration.

Note that as in the static model, firms' external financing capacity cannot exceed their collateral value, i.e., $q_t^K K_t$ in aggregate. The analysis so far has focused on the case that this constraint never binds. This assumption is satisfied by the calibrated solution: the ratio of bank loans to collateral value varies from 4.6% to 83.3% depending on the state of the world (i.e., η_t).[38]

Bank balance-sheet cycle. The state variable η_t measures the aggregate bank equity, i.e., the size of liquidity suppliers, relative to the size of firms (liquidity demanders). Bank equity affects the real economy through deposit creation. When η_t increases, banks expand balance sheets and issue more deposits, so firms can hold more liquidity for investment, and the economy grows faster. Figure 5 shows the statistical properties of η_t. Panel A plots several sample paths of η_t by simulating the law of motion (Equation (11)).[39] The paths are bounded by the issuance and payout.

Panel B of Figure 5 plots the impulse response function of η_t, showing that shocks have persistent effects. Because the law of motion of η_t is non-linear and state-dependent, we cannot define impulse response functions as in linear time series analysis. Thus, to illustrate the persistent impact of shocks, I fix the value of η_t to the median value under the stationary distribution, and consider an increase. The figure plots the percentage change of the term structure of expectation, i.e., the expected value of η_{t+T} with T ranging from one month to ten years.[40] Shock impact dissipates gradually. The initial increase of 11.6% raises the expected value in ten years by 2.4%.

The persistence is caused by banks' precautionary behavior, which is in turn due to the equity

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[38]The model will have two state variables, η_t and firm equity scaled by capital stock, if in some states of the world, firms' collateral constraint binds. This certainly enriches the model and improves its quantitative performance, but sacrifices the transparency of mechanisms. Rampini and Viswanathan (2017) study the joint dynamics of firm and intermediary equities (see also Elenev, Landvoigt, and Nieuwerburgh (2017)).

[39]One step is one day so the shock ΔZ_t is drawn from normal distribution $N(0, 1/365)$. Asmussen, Glynn, and Pitman (1995) show a weak order of convergence of 1/2 for discretized regulated diffusion.

[40]The expectation is calculated by the Kolmogorov backward equation (i.e. the Feynman–Kac formula), for reflected diffusion processes. The partial differential equations are solved by the Method of Lines (Schiesser and Griffiths (2009)). Borovička, Hansen, and Scheinkman (2014) provide an alternative, systematic framework to define and calculate impulse responses and the term structure of shock elasticity for non-linear diffusion processes.
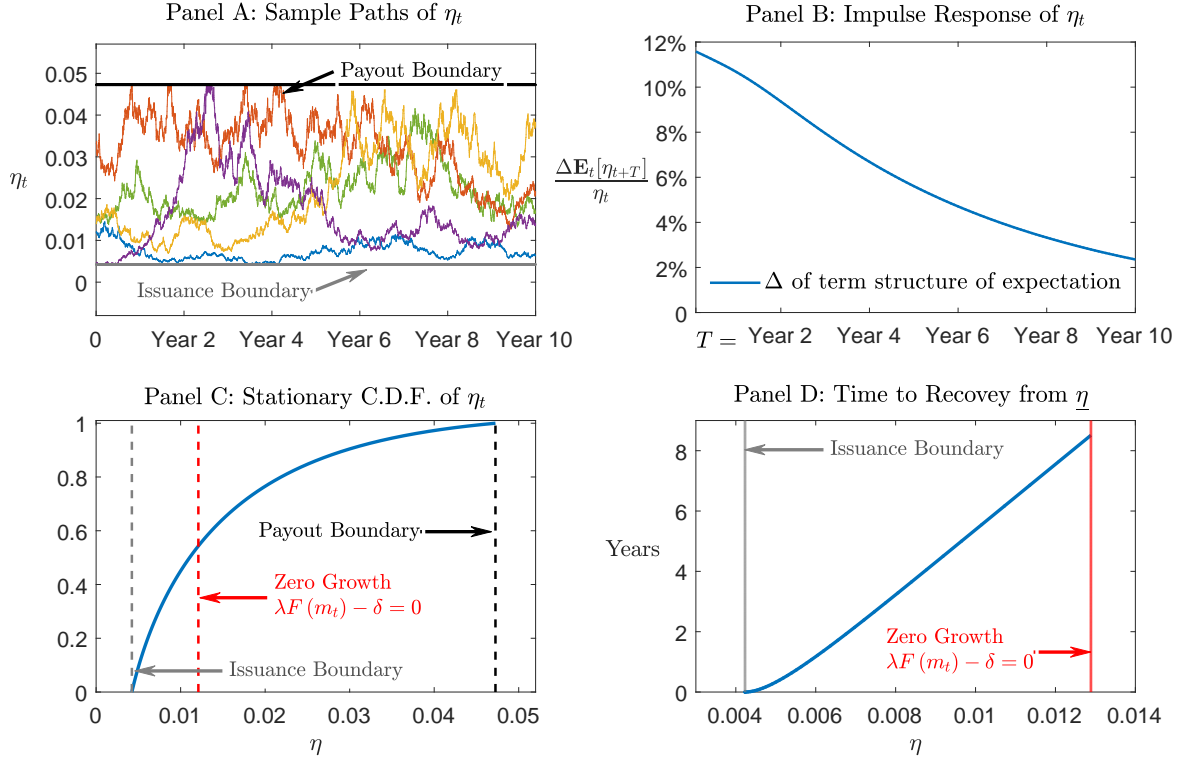
Figure 5: **State Variable Dynamics.** This figure shows the statistical properties of state variable η_t, the ratio of banking capital to illiquid, productive capital. Panel A plots the simulated paths of η_t. Panel B plots the percentage change of the expected value of η at different horizons in response to a positive shock. It shows the persistence of shock impact. Panel C plots the stationary cumulative distribution function (C.D.F.) of η_t that starts from the issuance boundary, passes the zero growth point, and ends at the payout boundary. Panel D plots expected years to reach a value of η (horizontal axis) when the current state is at the issuance boundary. It ends at the zero growth point.

issuance cost. If bankers could issue equity freely (i.e. $\chi = 0$), they no longer need to retain equity. Whenever q_t^B is above one, signaling an improvement of the investment opportunity set, bankers raise equity from households; whenever q_t^B is below one, bankers distribute dividends. The dilution cost implies that equity is only raised infrequently when q_t^B reaches $1 + \chi$, signaling severe capital shortage. χ opens up a wedge between q_t^B, the value of one dollar as banks' retained equity, and 1, the value of one dollar paid out as dividends. As a result, banks preserve a financial slackness. Unless the economy hits the payout boundary, banks accumulate equity.

Panel C shows the stationary cumulative probability function (c.d.f.) from Proposition 3. The curve starts from zero at the issuance boundary, and ends at one at the payout boundary. Around

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50% of the time, the economy is in a region with negative growth. I calibrate the mean growth rate to a relatively low number, 0.74% per year, which is the growth of output attributed to intangible investment from 1995 to 2007 (Corrado and Hulten (2010)). Intangible investment, such as R&D, relies heavily on internal liquidity, and thus, corresponds well to investment in the model.[41]

Panel D of Figure 5 plots the expected time to reach different values of η_t from the issuance boundary $\underline{\eta}$ (Proposition 3). The right bound marks the lowest value of η_t that delivers a non-negative economic growth rate. In expectation, it takes more than eight years to recover from the bottom of a recession. As previously discussed, stagnation results from bank's deleveraging in the bad states and the asymmetric impact of shocks on bank equity near the reflecting boundaries.

Procyclicality. As illustrated by Figure 4, the deposit market moves with $\left(\gamma_t^B, q_t^K\right)$, which in turn varies with the state variable, η_t. γ_t^B, the risk price that bankers charge for issuing safe deposits backed by risky loans, drives the liquidity supply, while q_t^K, the capital value, shifts firms' liquidity demand curve. Panel A of Figure 6 shows γ_t^B as a function of η_t. Because one unit of time is set to one year, γ_t^B is the annual Sharpe ratio of risky lending financed by risk-free deposits. γ_t^B decreases in η_t. When the economy is close to the bank recapitalization boundary $\underline{\eta}$, banks charge a price of risk close to 0.25; at the payout boundary, γ_t^B is zero. Good shocks increase η_t and decrease γ_t^B, shifting downward the bankers' indifference curve in Figure 4. Panel B shows that q_t^K increases in η_t. Even if the variation of q_t^K is not quantitatively large, it is sufficient to generate strong procyclicality of firms' liquidity demand that leads to procyclical bank leverage.[42]

Panel C plots the equilibrium liquidity premium, $\rho - r_t$, against the stationary cumulative probability of η_t. For instance, 0.2 on the horizontal axis is mapped to a liquidity premium equal to 36bps, meaning that 20% of the time, liquidity premium is larger than or equal to 36 basis points. The width of an interval on horizontal axis shows how much time the economy spends in

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[41]Since Hall (1992) and Himmelberg and Petersen (1994), it has been well documented that R&D heavily relies on internal financing (see Hall and Lerner (2009) for a survey on innovation financing). A difficulty of external financing is that the knowledge asset created by R&D is intangible, partly embedded in human capital, and is often specialized to the particular firm in which it resides. It is difficult for investors to repossess such intangible assets in case of default.

[42]As well documented in the empirical literature, asset price variation is dominated by the variation in discount rate (e.g. Cochrane (2011)). In the model, discount rate is fixed at ρ, so the variation in q_t^K is purely driven by firms' cost of liquidity management (liquidity premium), and their choice of liquidity holdings that determines the capital growth. Thus, quantitatively, we would not look for large variation in capital value over the cycle.
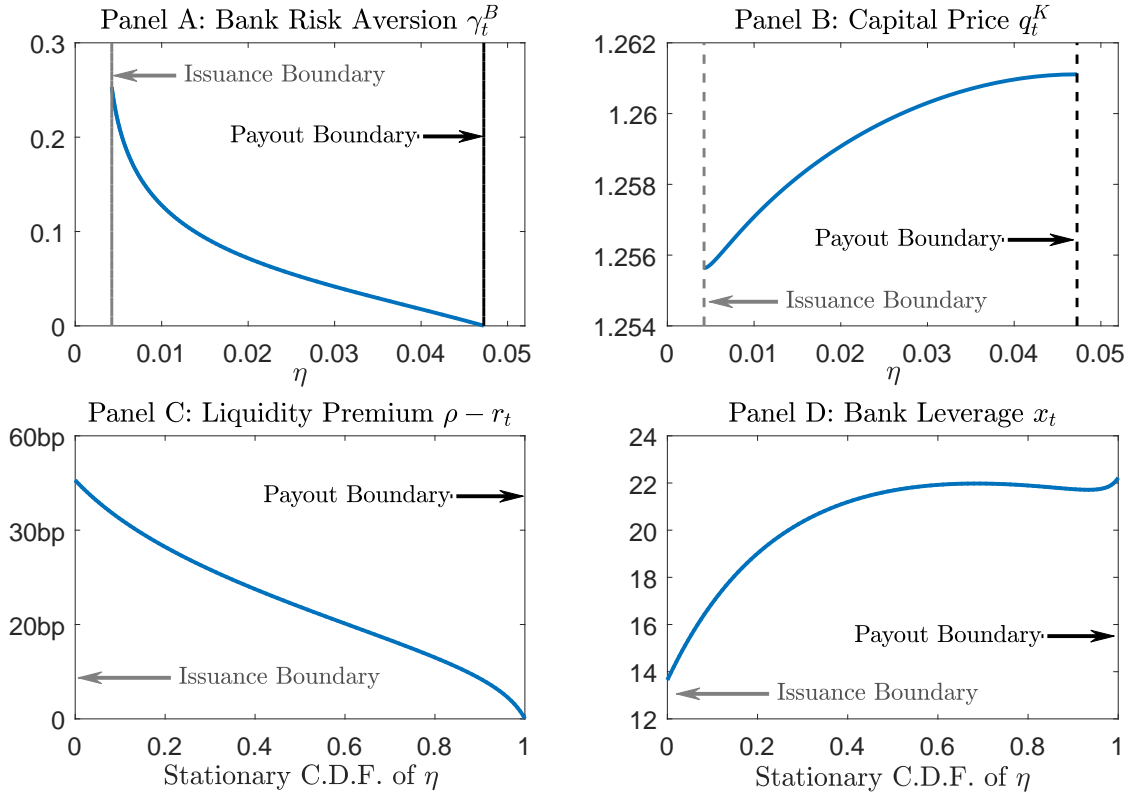
Figure 6: **Procyclical Leverage.** This figure plots bank risk price γ_t^B (Panel A) and capital value q_t^K (Panel B) against η_t. These two variables determine the locale of deposit market equilibrium (Figure 2). Panel C and D plot liquidity premium and bank book leverage respectively against the stationary cumulative distribution function (C.D.F.). They show how often (horizontal axis) the variable of interest stays in certain regions (vertical axis).

the region. The interval $[0.2, 0.3]$ is mapped to $[32\text{bps}, 36\text{bps}]$, so $30\% - 20\% = 10\%$ of the time, the liquidity premium is between 32bps and 36bps. Reading the graph from left to right, we follow a path of positive shocks, and see as the banking sector builds up equity, their risk-taking capacity expands, and thus, the liquidity premium, i.e., firms' cost of liquidity management, declines.

Panel D plots bank leverage, x_t, against the stationary cumulative probability of η_t. Leverage is mostly procyclical. Reading the graph from left to right, we see that when bank equity increases, banks issue even more deposits, so their leverage increases. The reason is that firm foresee a lower cost of liquidity management going forward, and thus, assign a higher valuation of capital, making the incentive stronger to hoard liquidity in case the investment opportunity arrives the very next instant (Figure 3 and 4). From right to left, we see how a crisis unfolds and banks deleverage.

There is a small region near the 80th percentile, where bank leverage rises following bad shocks (moving left). In reality, balance-sheet cyclicality may differ by the types of financial intermediaries. Adrian and Shin (2010) find the book leverage of broker-dealers is procyclical. He, Khang, and Krishnamurthy (2010) show that commercial banks' leverage increased in the 2007-09 crisis. Commercial banks' ability to issue deposits depends not only on equity as risk buffer but also regulatory constraints and government guarantees. Banks in the model are actually closer to shadow banks. Many have argued that the demand for money-like securities is one of the major drivers behind shadow banking developments (Gennaioli, Shleifer, and Vishny, 2013; Pozsar, 2014). The model does not have heterogeneous intermediaries, but the countercyclicality of leverage at high η_t reflects some complexity of the leverage cycle.

The procyclicality of bank leverage helps explain the statistical properties of the model in Figure 5. In Panel C, there is a relatively small probability of states with high banks' equity. In good times, banks' leverage is high, so the economy is sensitive to shocks. When the economy is close to the payout boundary, the impact of good shocks is limited, because large good shocks trigger dividend distribution, meaning that the banking sector cannot grow beyond $\bar{\eta}$. High leverage only serves to amplify the impact of negative shocks. Therefore, the downside risk accumulates as leverage rises. Because of this fragility, the economy spends less time in boom. Panel D of Figure 5 also shows the slow recovery. When the economy is close to the issuance boundary, the impact of negative shocks is bounded. Low bank leverage only serves to reduce the impact of good shocks on bank equity, so banks accumulate equity slowly and the economy tends to get stuck in recessions. Countercyclical leverage would have led to the exactly opposite pattern.[43]

Static and dynamic inefficiencies. In Corollary $1'$, the liquidity premium is equal to bankers' required risk compensation $\gamma_t^B \sigma$. A higher γ_t^B, and thus, a higher liquidity premium, directly translates into a larger gap between the cash-constrained investment and the current target i_t^*, defined by $q_t^K F'(i_t^*) = 1$. Panel A of Figure 7 shows the static inefficiency, measured by the percentage

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[43]Models with countercyclical leverage generate instability through other mechanisms, such as fire sale in He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014), which need intermediaries to hold long-term assets, and thereby, are exposed to endogenous volatility of asset prices. To highlight the novel link between leverage and liquidity demand, I shut down this channel by restricting banks' investment to short-term loans.
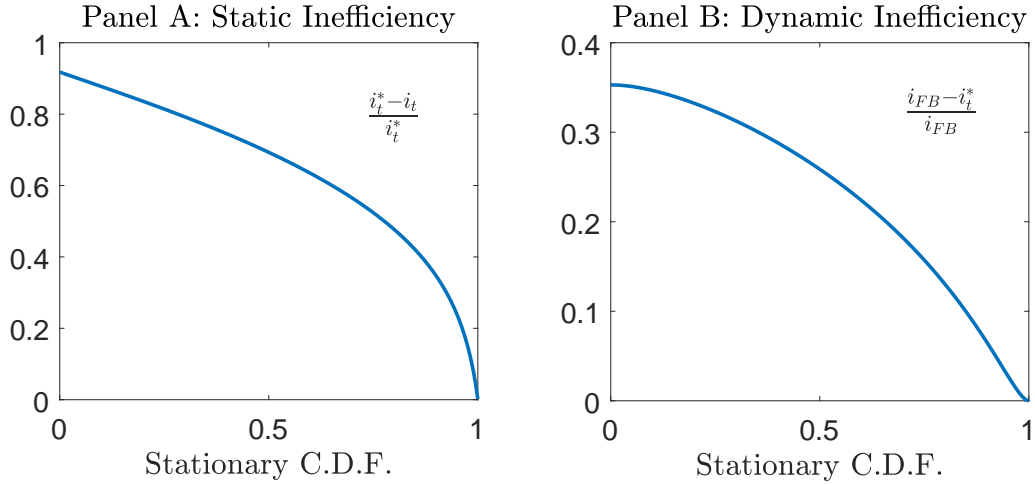
Figure 7: **Static and Dynamic Investment Inefficiencies.** This figure plots static investment inefficiency (Panel A), measured by the percentage deviation of equilibrium investment rate (i_t) from the target rate implied by the equilibrium capital value (i_t^*), and dynamic investment inefficiency (Panel B), measured by the percentage deviation of target rate (i_t^*) from the first-best investment rate (i_{FB}), against the stationary cumulative distribution function (C.D.F.). The plots show how often (horizontal axis) the variable of interest stays in certain regions (vertical axis).

deviation of the actually investment rate from the target, i_t^*. As bankers' risk capacity increases, the liquidity premium declines, and firms hold more deposits for investment. Moving from the left to the right, the investment wedge declines from 95% at the depth of recession to 0% at $\overline{\eta}$.

Panel B of Figure 7 shows the dynamic inefficiency, measured by the percentage deviation of the current target from the first-best investment rate defined by Equation (18). The wedge varies with capital value, and declines as η_t increases, because q_t^K increases in η_t (as shown in Panel B of Figure 6). Around 50% of the time, the current target is 25% or more below the first-best level.

Investment inefficiencies decrease welfare. Given the constant productivity α, the aggregate consumption is determined by the capital stock K_t. Under risk-neutral preference, what matters, from a welfare perspective, is the expected growth rate of K_t, i.e., $\lambda F(m_t) - \delta$, newly created capital net of destroyed existing capital. Therefore, through firms' liquidity constraint on investment, welfare is tied to liquidity creation. Insufficient liquidity suggests the government has a role in supplying liquidity. Does public liquidity really improve welfare? How will banks respond? Will public liquidty stabilize the economy? The next section aims to answer these questions.

4 Public Liquidity: Instability, Stagnation, and Welfare

It has long been recognized that government debt offers monetary services (Patinkin (1965); Friedman (1969)). Repo market developments since 1980s enhanced the liquidity of Treasury securities (Fleming and Garbade (2003)). In this section, I introduce government debt as liquidity, an alternative to bank debt. Whether government alleviates the liquidity shortage faced by firms depends on how banks react. The competition between intermediated and public liquidity can *destabilize* the economy by amplifying bank leverage cycle, and thereby, exacerbate investment inefficiencies. These findings complement the literature on government debt as a means to financial *stability* (Greenwood et al. (2015); Krishnamurthy and Vissing-Jørgensen (2015); Woodford (2016)).

Setup. Firms can hold bank debt or government debt to relax the liquidity constraint on investments. Thus, issued at t, government debt pays the same risk-free rate as deposits, $r_t dt$, at $t + dt$. To focus on the liquidity provision role of government debt, I abstract away other fiscal distortions: issuance proceeds are distributed as lump-sum payments and debt is repaid with lump-sum tax on households. Moreover, I assume the government faces a debt limit that is proportional to the scale of the economy, i.e., $M^G K_t$, and consider a debt management strategy in line with Friedman's rule – the government always issues the maximum amount when the liquidity premium is positive.[44]

Firms' liquidity holdings per unit of capital are now $m_t + M^G$. Substituting it into the optimality condition of deposit holdings in Lemma 1′, we have a new deposit demand curve:

$$\rho - r_t = \lambda \left[q_t^K F' \left(m_t + M^G \right) - 1 \right].$$

Because $F \left(\cdot \right)$ is a concave function, the demand curve is shifted inward. Introducing government debt reduces the marginal benefit of deposit holdings (m_t), and the equilibrium liquidity premium. By helping firms manage liquidity, government debt is likely to have a positive effect on investment and growth, which is similar to the investment crowding-in effect in Woodford (1990b) and

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[44]Friedman's rule says individuals' opportunity cost to hold liquidity should be equal to the social cost of creating liquidity (Friedman (1969); Woodford (1990a)). Firms' opportunity cost to hold liquidity is $\rho - r_t$, the liquidity premium. The private sector's marginal cost of liquidity creation is $\gamma_t^B \sigma$, the risk compensation charged by bankers. In contrast, the government's cost of liquidity creation is zero as I already assume away any fiscal distortions. Therefore, Friedman's rule suggests that the government should maximize its debt issuance (i.e., liquidity supply). However, as will be discussed later, because public liquidity crowds out intermediated, Friedman's rule is not necessarily optimal.

Holmström and Tirole (1998).[45] However, the actual effect depends on how banks react.

This setup does not distinguish Treasury securities from central bank liabilities that pay interests. It intends to capture public liquidity supply from the traditional expansion of monetary base and from government issuing liquid securities. Accordingly, the setup has an alternative interpretation. Government debt is held by a central bank, who issues an equal amount of reserves that pay the same interest rate. Reserves are held by banks who in turn issue an equal amount of deposits to firms. On this chain, both the central bank and banks are just pass-throughs. Intermediating between risk-free assets and liabilities does not require additional bank equity. In reality, the line between short-term Treasury securities and reserves is becoming increasingly blurry.[46] Interest on reserves has been introduced in countries such as the U.K. and the U.S. Federal Reserve now allows the public to hold reserves through money market funds that are reverse repo counterparties.

Leverage cycle and instability. Figure 8 compares the model's performances when the government debt-to-output ratio equals to 0%, the benchmark case, and 50%.[47] Panel A plots the liquidity premium, $\rho - r_t$, against the state variable, η_t. Government debt reduces the liquidity premium, which is in line with the evidence (Krishnamurthy and Vissing-Jørgensen (2012); Greenwood and Vayanos (2014); Greenwood, Hanson, and Stein (2015); Sunderam (2015)). However, by raising r_t, public liquidity increases banks' debt cost, and thus, reduces their return on equity. In response, banks reset payout and issuance policies, shifting both boundaries to the left. This *bank equity crowding-out* effect is also shown in Panel B, the stationary cumulative distribution of η_t.[48]

Panel C of Figure 8 shows that government debt makes endogenous risk accumulate faster in booms (i.e., as η_t increases). Endogenous risk is measured by σ_t^η, the instantaneous shock elasticity of η_t. σ_t^η is plotted against the stationary cumulative probability of η_t, so we can compare the two

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[45]The traditional crowding-out effect describes how the government debt supply raises interest rate in general, and thereby crowds out private investment through a higher financing cost. My model does not entertain this effect, because by shutting down external financing completely, the model has cash being the only determinant of investment.

[46]The liquidity coverage ratio (Basel Committee on Bank Supervision (2013)), which counts government securities as banks' liquidity holdings, is introduced as a modern version of reserves requirement.

[47]Comparative statics show the model's performances in response to *unexpected* and *permanent* changes in the government debt supply. The current practice of U.S. Treasury debt management emphasizes predictability (Garbade (2007)), but since the financial crisis, there has been a considerable amount of uncertainty on fiscal policies, especially on the debt level (e.g. Baker, Bloom, and Davis (2015); Kelly, Pástor, and Veronesi (2016))

[48]The stationary distribution is calculated using Proposition 4 using the model solutions under different M^G.
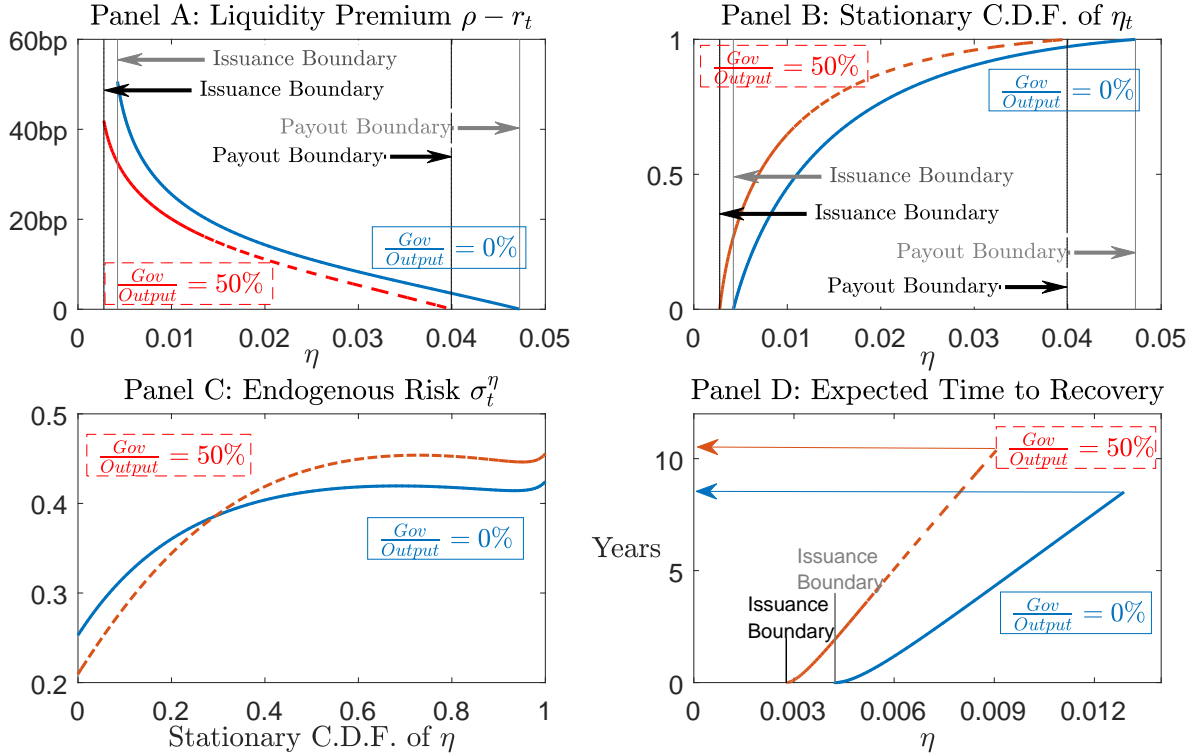
Figure 8: **Public Liquidity: Instability and Stagnation.** This figure plots variables of the benchmark model without government debt and model with 50% government debt-to-output ratio (dotted line). Panel A plots liquidity premium $\rho - r_t$ against state variable η_t. Panel B plots the stationary C.D.F. of η_t. Panel C plots endogenous risk, σ_t^η, i.e, the instantaneous standard deviation of η_t against the stationary C.D.F. Panel D shows expected years to reach a value of η_t (horizontal axis) when the current state is at the issuance boundary (ending at the zero growth point).

models in the corresponding phases of their cycles. Reading the graph from left to right, we see σ_t^η increases faster when government debt-to-output ratio is 50%.

Endogenous risk is directly linked to leverage, $\sigma_t^\eta = (x_t - 1)\,\sigma$ (Equation (11)), so Panel C also shows that government debt amplifies the bank leverage cycle. Stronger leverage procyclicality makes the economy more sensitive to shocks in good times (high η_t) and less sensitive to shocks in bad times (low η_t). As previously discussed, shock impact is asymmetric near the boundaries, so the economy gets stuck near $\underline{\eta}$ and very responsive to bad shocks near $\overline{\eta}$. As a result, probability mass is shifted towards low η_t states, as shown in Panel B (i.e., a more concave c.d.f.).

To understand how public liquidity amplifies the leverage cycle, consider the economy at the issuance boundary $\underline{\eta}$. As shown in Panel A of Figure 8, government debt reduces the liquidity

premium at $\underline{\eta}$, so banks' risk price γ_t^B must decline so that their indifference condition, $\rho - r_t = \gamma_t^B \sigma$, holds. By Itô's lemma, we can decompose risk price (see also Equation (19)): $\gamma_t^B = -\sigma_t^B = -\epsilon_t^B \sigma_t^\eta$, where $\sigma_t^\eta = (x_t - 1)\sigma$ is the instantaneous shock elasticity of η_t. The boundary condition (3) in Proposition 4 implies that $\epsilon_t^B = -1$ at $\underline{\eta}$, so to reduce banks' risk price, the state variable η_t needs to be less volatile (i.e., a lower σ_t^η), meaning that the equilibrium bank leverage x_t has to be lower. Therefore, public liquidity reduces bank leverage at the issuance boundary $\underline{\eta}$.

However, due to equity issuance cost, government debt increases banks' leverage *away* from $\underline{\eta}$, and thereby, amplifies leverage procyclicality. The wedge between q_t^B and 1 measures future profits (return on equity) that come from leverage, i.e., financing loans with deposits, and the liquidity premium. At $\underline{\eta}$, this wedge is χ, as it must compensate the issuance cost. When the liquidity premium is squeezed by government in every state (Panel A of Figure 8), banks' leverage has to increase on future equilibrium paths to sustain this level of profits. Thus, while reducing banks' leverage at $\underline{\eta}$, public liquidity increases leverage where $\eta_t > \underline{\eta}$, amplifying procyclicality.

Greenwood, Hanson, and Stein (2015), Krishnamurthy and Vissing-Jørgensen (2015), and Woodford (2016) also explore the financial stability implications of government debt when it serves as a substitute for intermediaries' debt. A common prediction is that by squeezing the liquidity premium, government debt crowds out bank debt, and thereby, decreases banks' leverage and stabilizes the economy.[49] What is missing is banks' dynamic equity management under frictions.

My model also highlights the competition between bank and government debt in the money market, but it makes two unique predictions regarding the destabilizing effect of government debt. By crowding out banks' profit, government debt crowds out bank equity. It also amplifies the bank leverage cycle, as banks try to sustain a level of return on equity that compensates issuance costs.

Stagnation. Panel D of Figure 8 plots the recovery paths from the bank issuance boundary. The Y-axis shows the expected number of years it takes to travel from $\underline{\eta}$ to different values of η on the X-axis. For instance, when the government debt-to-output ratio is 0%, it takes more than one year to reach $\eta = 0.006$. Both curves end at the recovery point, which is the lowest value of η that has a non-negative economic growth rate, i.e., $\lambda F\left(m_t + M^G\right) - \delta \geq 0$.

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[49]Bank debt crowding-out effect of government debt has been documented by Bansal, Coleman, and Lundblad (2011), Greenwood, Hanson, and Stein (2015), Krishnamurthy and Vissing-Jørgensen (2015), and Sunderam (2015).
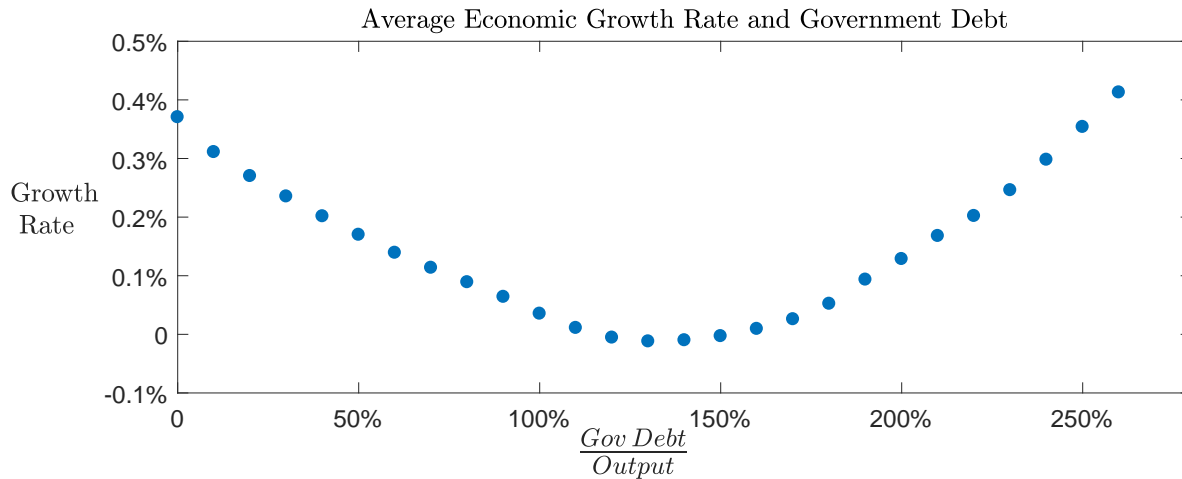
Figure 9: **Non-monotonic Effects of Public Liquidity on Investment and Growth.** The figure plots the mean growth rate (based on stationary distribution) for models with different levels of government debt-to-output ratio.

Government debt prolongs recessions. Raising government debt-to-output ratio from 0% to 50% delays the recovery by two years. The reason is that because firms still rely on banks as the *marginal* supplier of liquidity, the profit crowding-out effect delays the recovery of banks. Lower return on equity slows down the accumulation of bank equity. Admittedly, more government debt also makes banks less relevant, because firms already hold government, and thus, the marginal value of bank debt declines. In the extreme case where the government has a large debt capacity to satiate firms' liquidity demand, the economy grows with the first-best investments.

Growth and welfare. Figure 9 shows the mean growth rate under stationary distribution for different levels of government debt. Since the capital productivity is constant and agents are indifferent about consumption timing, the mean growth rate of K_t proxies welfare. Before the government debt-to-output ratio reaches around 130%, more government debt decreases welfare, because *on average*, one dollar more public liquidity crowds out more than one dollar of intermediated liquidity. It might seem difficult to reconcile this with Panel A of Figure 8, which shows more government debt decreases the liquidity premium in *every* state of the world, and thus, must raise firms' investment rate in *every* state of the world. The key lies in the shift of probability distribution.

Public liquidity amplifies the bank leverage cycle and shifts the probability mass towards states where banks are relatively undercapitalized and intermediaries' liquidity supply is weak

(Panel B of Figure 8). Therefore, even if every state of the world has a higher growth rate, the average growth rate can decrease when more probability is assigned to bad states. However, when government debt almost satiates firms' liquidity demand, bank equity and intermediated liquidity become less relevant. Once passing the point of 130% government debt-to-output ratio, government debt improves welfare.

The decreasing leg in Figure 9 is particularly relevant for understanding the U.S. economy around the Great Recession. From 2001 to 2008, the public debt-to-GDP ratio rose from c.55% to c.70%. This coincided with a period of strong leverage procyclicality in the financial sector. Many argue that downside risk accumulated as a result (FSB (2009)). The post-crisis period saw an even more dramatic increase in government debt, partly due to quantitative easing. By 2012, the public debt-to-GDP ratio had reached its current level, c.100%. Meanwhile, economic recovery was slow.

5   Conclusion

This paper studies the liquidity provision function of financial intermediaries. To buffer liquidity shocks, firms hold bank debt. Banking crises cause the contraction of liquidity supply, which compromises firms' liquidity management. The frequency and duration of crisis depend on the bank leverage cycle and banks' payout and equity issuance decisions. Government debt as an alternative source of liquidity may contribute to financial instability. By squeezing banks' profits from liquidity creation, government debt crowds out bank equity and amplifies the leverage cycle, and thus, may reduce welfare. The key to this destabilizing effect is banks' dynamic equity management under equity issuance frictions, which has been ignored by the pioneer works in the literature.

In the model, the government walks on a tightrope. Increasing government debt temporarily alleviates the liquidity shortage that firms face, but in the long run, by crowding out bank equity and amplifying bank leverage cycle, it can crowd out more intermediated liquidity, causing more severe liquidity shortage. This trade-off suggests that optimal strategy of public liquidity supply is macroprudential, i.e., contingent on the relative tightness of banks' and firms' financial constraints.

In this paper, public liquidity crowds out intermediated liquidity. In a richer environment,

public liquidity can crowd in intermediated liquidity. Banks hold government debt to buffer their own liquidity shocks, so by relaxing banks' liquidity constraint, government debt allows banks to expand balance sheet and issue more deposits. Government debt can also finance the recapitalization of banks, preventing a sudden evaporation of intermediated liquidity due to banks' default. Benigno and Robatto (2019) study how fiscal capacity allows such government interventions.

The demand for money-like securities, or safe assets in general, has attracted enormous attention. Theoretical studies in the literature devote tremendous effort in characterizing the issuers, intermediaries for instance, while relegate the modeling of investors who demand money-like or safe assets, often by assuming safety in utility. This paper takes a step forward, relating money demand to corporate liquidity management. Other sources of demand, such as foreign sovereigns and institutional investors, should also be modeled carefully. The endogenous and distinct dynamics of different money market investors is as important as the behavior of issuers (financial intermediaries), when it comes to understand the causes and consequences of financial crisis.

Appendix I   Proofs

I.1   Static Model

Firms' problem. Let $\underline{k}_0(s)$ denote capital endowments of a firm s at $t = 0$, and $k_0(s)$ its capital *demand*. Aggregate stock K_0 is $\int_{s\in[0,1]} \underline{k}_0(s)\, ds$. Capital market clears, so $K_0 = \int_{s\in[0,1]} k_0(s)\, ds$. To save notations, I suppress firm index s. Given q_0^K, a firm's wealth is $w_0 = q_0^K \underline{k}_0$. At $t = 0$, a firm chooses capital k_0, deposits per unit of capital m_0, consumption c_0, the value of bank loan l_0, and funds raised by issuing securities to households h_0, and at $t = 1$, chooses investment rate i_1.

By promising to *expected* payments of v_1^H at $t = 1$, a firm raises h_0 from households at $t = 0$. Given household discount rate ρ, a competitive security market implies $v_1^H = h_0(1 + \rho)$.

Let v_0 denote the firm's value function, which is equal to $\max\limits_{c_0 \geq 0, h_0 \geq 0} c_0 + \frac{1}{1+\rho}\left(v_1^* - v_1^H\right)$, where v_1^* is the *maximized expected* value before repaying households, a function of other optimal choices (k_0, m_0, l_0, i_1). Substituting $v_1^H = h_0(1 + \rho)$ into the expression, we have

$$v_0 = \max_{c_0 \geq 0, h_0 \geq 0} c_0 - h_0 + \frac{1}{1+\rho} v_1^*.$$

Therefore, what matters is the net consumption $(c_0 - h_0)$ or net financing $(h_0 - c_0)$. Going forward, we allow c_0 to take positive or negative values. When $c_0 > 0$, the entrepreneur consumes, and when $c_0 < 0$, the entrepreneur raises funds from households. We can write the firms' value function as

$$v_0 = \max_{c_0^B \in \mathbb{R}} c_0 + \frac{1}{1+\rho} v_1^*.$$

The firm can also issue securities to banks. Let v_1^B denote the firm's *expected* repayment to banks at $t = 1$. Note that in the analysis of the firm's problem, we do not need to specify the contractual form of securities issued to banks or households. All that matters is the expected repayment. Let v_1^{**} denote the *maximized expected* value of the firm before repaying both households and banks, so $v_1^{**} = v_1^* + v_1^B$ by definition. We can rewrite the value function as

$$v_0 = \max_{c_0^B \in \mathbb{R}} c_0 + \frac{1}{1+\rho}\left(v_1^{**} - v_1^B\right).$$

Because bank debt earns a liquidity premium, the required rate of return of banks can be different from that of households. Let ρ_0^B denote banks' required expected return in equilibrium. For the

firm to raise l_0 from banks at $t = 0$, it must deliver an expected repayment to competitive banks equal to $v_1^B = l_0 \left(1 + \rho_0^B\right)$. Thus, we can rewrite the value function as

$$v_0 = \max_{c_0^B \in \mathbb{R}, l_0 \geq 0} c_0 - \frac{1 + \rho_0^B}{1 + \rho} l_0 + \frac{1}{1 + \rho} v_1^{**}, \tag{20}$$

where v^{**} is a function of other optimal choices (k_0, m_0, i_1). If $\rho_0^B < \rho$, the firm only issue securities to banks; likewise, if $\rho < \rho_0^B$, the firm only issue securities to households if at all. Since we study an equilibrium where firms do borrow from banks, it must be true that $\rho_0^B \leq \rho$.

The firm's financing capacity depends on its pledgeable value at $t = 1$. Newly created capital is not pledgeable, and a fraction $\delta - \sigma Z_1$ of existing capital will be gone by $t = 1$ (with $\mathbb{E}_0 [Z_1] = 0$), so the expected pledgeable value is $\alpha k_0 (1 - \delta)$. The firm faces the following financing constraint:

$$l_0 \left(1 + \rho_0^B\right) + \mathbb{I}_{\{c_0 < 0\}} \left(-c_0\right) \left(1 + \rho\right) \leq \alpha k_0 \left(1 - \delta\right), \tag{21}$$

where $\mathbb{I}_{\{.\}}$ is an indicator function. Note that when $-c_0 < 0$, the firm raises $|c_0|$ from households.

The firm also faces the budget constraint and the liquidity constraint:

$$c_0 + q_0^K k_0 + m_0 k_0 \leq w_0 + l_0, \text{ where } c_0 \in \mathbb{R}, \tag{22}$$

$$i_1 \leq m_0. \tag{23}$$

Given capital value q_0^K, the deposit rate r_0, and the required expected return of banks ρ_0^B, the firm maximizes the objective in Equation (20) subject to constraints (21), (22), and (23), with the expected total firm value (before repayment to investors) at $t = 1$ given by

$$v^{**} = \underbrace{\alpha k_0 \left(1 - \delta\right)}_{\text{expected surviving capital value}} + \underbrace{\left(1 + r_0\right) m_0 k_0}_{\text{deposits}} + \underbrace{\lambda \left(\alpha F \left(i_1\right) - i_1\right) k_0}_{\text{expected new capital net value}}. \tag{24}$$

Let κ_0, ψ_0, and θ_0 denote the Lagrange multipliers of the financing, budget, and liquidity constraints respectively. We can write the Lagrange (omitting the non-negativity constraints):

$$
\begin{aligned}
v_0 = \max_{c_0^I \in \mathbb{R}, l_0 \geq 0, k_0 \geq 0, m_0 \geq 0, i_1 \geq 0} \quad & c_0 - \frac{1 + \rho_0^B}{1 + \rho} l_0 + \frac{1}{1 + \rho} \left[\alpha k_0 \left(1 - \delta\right) + \left(1 + r_0\right) m_0 k_0 \right. \\
& \left. + \lambda \left(\alpha F \left(i_1\right) - i_1\right) k_0\right] + \theta_0 \left(m_0 - i_1\right) \\
& + \kappa_0 \left[\alpha k_0 \left(1 - \delta\right) - l_0 \left(1 + \rho_0^B\right) - \mathbb{I}_{\{c_0 < 0\}} \left(-c_0\right) \left(1 + \rho\right)\right] \\
& + \psi_0 \left(w_0 + l_0 - c_0 - q_0^K k_0 - m_0 k_0\right).
\end{aligned}
$$

Proof of Lemma 2. First, we solve ψ_0. We must have the coefficient of c_0 equal to zero

$$1 + \kappa_0 \mathbb{I}_{c_0 < 0} (1 + \rho) - \psi_0 = 0, \tag{25}$$

Because c_0 can take either positive or negative values. In equilibrium, banks lend out at least some of their goods endowments to firms to carry net worth to $t = 1$. Because goods cannot be stored, in aggregate, entrepreneurs must consume, so $c_0 > 0$, so from Equation (25), $\psi_0 = 1$.

Next, we solve κ_0, the shadow value of financing. In equilibrium $l_0 > 0$ (i.e., not a corner solution), so locally the entrepreneur must be indifferent. Thus, the coefficient of l_0 is equal to zero

$$-\frac{1 + \rho_0^B}{1 + \rho} - \kappa_0 \left(1 + \rho_0^B\right) + \psi_0 = 0. \tag{26}$$

Substituting $\psi_0 = 1$ into Equation (26), we have

$$\kappa_0 = \frac{1}{1 + \rho_0^B} - \frac{1}{1 + \rho}. \tag{27}$$

If the firm promises one unit of goods in expectation at $t = 1$, it can obtain $\frac{1}{1+\rho_0^B}$ bank financing and $\frac{1}{1+\rho}$ household financing at $t = 0$. Intuitively, κ_0 is the difference between the price of securities issued to banks and the price of securities issued to households. $\kappa_0 > 0$ if and only if $\rho_0^B < \rho$.

Multiplying both sides of Equation (27) by $\left(1 + \rho_0^B\right)(1 + \rho)$, we have

$$\kappa_0 \left(1 + \rho_0^B\right)(1 + \rho) = \rho - \rho_0^B$$

Expanding the left hand side we have, $\kappa_0 + \kappa_0 \rho_0^B + \kappa_0 \rho + \kappa_0 \rho_0^B \rho$. The current time interval is 1, but if we let Δ denote the length of time between date 0 and 1, the product terms on the left-hand side are of the order of Δ^2 or higher. As Δ shrinks to zero, these terms approach to zero at a faster pace. To approximate the continuous-time expression, we ignore those product terms, so we have

$$\kappa_0 = \rho - \rho_0^B = \rho - (R_0 - \delta). \tag{28}$$

Proof of Lemma 1. The firm's first order condition with respect to m_0 is

$$\frac{1}{1 + \rho} (1 + r_0) k_0 - \psi_0 k_0 + \theta_0 = 0. \tag{29}$$

The first order condition with respect to i_1 is

$$\frac{1}{1 + \rho} \lambda \left(\alpha F'(i_1) - 1\right) k_0 - \theta_0 = 0. \tag{30}$$

Summing up Equation (29) and (30), we have

$$\frac{1}{1+\rho}(1+r_0)k_0 - \psi_0 k_0 + \frac{1}{1+\rho}\lambda(\alpha F'(i_1) - 1)k_0 = 0. \tag{31}$$

We focus on the situation where the investment technology $F(\cdot)$ is so productive that the liquidity constraint always binds, so $i_1 = m_0$. Substituting $\psi_0 = 1$ and rearranging the equation, we have

$$r_0 - \rho + \lambda(\alpha F'(m_0) - 1) = 0. \tag{32}$$

Proof of Lemma 3. The expected return on bank equity is:

$$x_0(1 + R_0 - E[\pi(Z_1)]) - (x_0 - 1)(1 + r_0) = 1 + r_0 + x_0(R_0 - E[\pi(Z_1)] - r_0).$$

Note that $E[\pi_D(Z_1)] = \delta$. When $Z_1 = -1$, the realized return on bank equity is:

$$1 + r_0 + x_0(R_0 - \delta - \sigma - r_0).$$

The representative bank starts with equity e_0 and has the following value function:

$$v(e_0; R_0, r_0) = \max_{y_0 \geq 0, x_0 \geq 0} y_0 e_0 + \frac{e_0(1 - y_0)}{(1 + \rho)}\{1 + r_0 + x_0(R_0 - \delta - r_0)$$
$$+ \xi_0[1 + r_0 + x_0(R_0 - \delta - \sigma - r_0)]\},$$

where y_0 is the banker's consumption-to-wealth ratio. The first order condition for x_0 is:

$$R_0 - r_0 = \delta + \gamma_0^B \sigma, \tag{33}$$

where $\gamma_0^B = \frac{\xi_0}{1+\xi_0} \in [0, 1)$ because $\xi_0 \geq 0$. Rearranging the equation, we have γ_0^B equal to the Sharpe ratio of loans: $\gamma_0^B = \frac{R_0 - \delta - r_0}{\sigma}$. When $\gamma_0^B > 0$, the capital adequacy constraint binds. Substituting the F.O.C. for x_0 into the value function, we have: $v(e_0; R_0, r_0) = y_0 e_0 + q_0^B(e_0 - y_0 e_0)$, where $q_0^B = \frac{(1+r_0)(1+\xi_0)}{(1+\rho)}$. The bank chooses $y_0 > 0$ only if $q_0^B \leq 1$.

Proofs of Proposition 1 and Corollary 1. Proofs are provided in the main text.

I.2 Continuous-time Model

Firms' problem: proof of Lemma 1′, Lemma 2′, and Proposition 2. Entrepreneurs ("firms") maximize life-time utility, $\mathbb{E}\left[\int_{t=0}^{+\infty} e^{-\rho t} dc_t\right]$, subject to the following wealth (equity) dynamics:

$$dw_t = -dc_t + \mu_t^w w_t dt + \sigma_t^w w_t dZ_t + (\widehat{w}_t - w_t) \, dN_t,$$

$\mu_t^w w_t$ and $\sigma_t^w w_t$ are the drift and diffusion terms that depend on choices of capital and deposit holdings and will be elaborated later. dN_t is the increment of the idiosyncratic counting (Poisson) process. $dN_t = 1$ if an investment opportunity arrives. At the Poisson time, firm equity jumps to

$$\widehat{w}_t = w_t + q_t^K F(m_t) k_t - m_t k_t.$$

We may conjecture that the value function is linear in equity w_t: $V_t = \zeta_t w_t$, where ζ_t is the marginal value of equity, and in equilibrium, follows a diffusion process:

$$d\zeta_t = \zeta_t \mu_t^\zeta dt + \zeta_t \sigma_t^\zeta dZ_t,$$

where $\zeta_t \mu_t^\zeta$ and $\zeta_t \sigma_t^\zeta$ are the drift and diffusion terms respectively. Note that firms' marginal value of wealth, ζ_t, is a summary statistic of firms' investment opportunity set, which depends on the overall industry dynamics, so it does not jump when an individual firm is hit by investment opportunities.

Under this conjecture, the Hamilton-Jacobi-Bellman (HJB) equation is

$$\rho V_t dt = \max_{dc_t \in \mathbb{R}, k_t \geq 0, m_t \geq 0, l_t \geq 0} dc_t - \zeta_t dc_t + \{w_t \zeta_t \mu_t^\zeta + w_t \zeta_t \mu_t^w + w_t \zeta_t \sigma_t^\zeta \sigma_t^w + \lambda \zeta_t [\widehat{w}_t - w_t]\} dt.$$

By the same logic in the analysis of firms' problem in the static model, firms' negative consumption is equivalent to raising funds from households. Firms can choose any $dc_t \in \mathbb{R}$, so ζ_t must be equal to one, and thus, I have also confirmed the value function conjecture.

Since ζ_t is a constant equal to one, μ_t^ζ and σ_t^ζ are both zero. The HJB equation is simplified:

$$\rho V_t dt = \max_{k_t \geq 0, m_t \geq 0, l_t \geq 0} \mu_t^w w_t dt + \lambda \left[q_t^K F(m_t) - m_t\right] k_t dt. \tag{34}$$

Equity drift has production, value change of capital holdings, deposit return, and loan repayment:

$$\mu_t^w w_t dt = \alpha k_t dt + \mathbb{E}_t \left(q_{t+dt}^K k_{t+dt} - q_t^K k_t\right) + r_t m_t k_t dt - l_t (R_t - \delta) \, dt$$

Let $d\psi_t$ denote the Lagrange multiplier of the budget constraint, $q_t^K k_t + m_t k_t \leq w_t + l_t$. The first-order condition (F.O.C.) for optimal deposit holdings per unit of capital is: $m_t \geq 0$, and

$$m_t \left\{ r_t dt + \lambda \left[q_t^K F'(m_t) - 1 \right] dt - d\psi_t \right\} = 0.$$

A fraction $(\delta dt - \sigma dZ_t)$ of capital is to be destroyed, so the capital evolves as

$$k_{t+dt} = k_t - (\delta dt - \sigma dZ_t) k_t.$$

Given the equilibrium capital value dynamics, $dq_t^K = q_t^K \mu_t^K dt + q_t^K \sigma_t^K dZ_t$, we have

$$q_{t+dt}^K k_{t+dt} - q_t^K k_t = q_t^K k_t \left[-(\delta dt - \sigma dZ_t) + \mu_t^K dt + \sigma_t^K dZ_t + \sigma \sigma_t^K dt \right].$$

The F.O.C. for optimal capital holdings is : $k_t \geq 0$, and

$$k_t \left\{ \alpha dt + q_t^K \left(-\delta + \mu_t^K + \sigma \sigma_t^K \right) dt + r_t m_t dt + \lambda dt \left[q_t^K F(m_t) - m_t \right] - \left(q_t^K + m_t \right) d\psi_t \right\} = 0.$$

The F.O.C. for optimal borrowing from banks is: $l_t \geq 0$, and

$$-(R_t - \delta) dt + d\psi_t = 0.$$

Finally, we have the complementary slackness condition: $d\psi_t \geq 0$, and

$$\left(w_t + l_t - q_t^K k_t - m_t k_t \right) d\psi_t = 0.$$

Substituting these optimality conditions into the HJB equation, we have

$$\rho V_t dt = w_t d\psi_t.$$

Because $\zeta_t = 1$, $V_t = w_t$, and $d\psi_t = \rho dt$. Substituting $d\psi_t = \rho dt$ into the F.O.C. for m_t, we have

$$r_t + \lambda \left[q_t^K F'(m_t) - 1 \right] = \rho.$$

Substituting $d\psi_t = \rho dt$ into the F.O.C. for k_t and rearranging the equation, we have

$$q_t^K = \frac{\alpha - (\rho - r_t) m_t + \lambda \left[q_t^K F(m_t) - m_t \right]}{\rho - (\mu_t^K - \delta + \sigma \sigma_t^K)}.$$

Substituting $d\psi_t = \rho dt$ into the F.O.C. for l_t, we have

$$R_t = \rho + \delta.$$

Banks' problem: proof of Lemma 3′. Conjecture that the bank's value function takes the linear

form: $v_t = q_t^B e_t$. In equilibrium, the marginal value of equity, q_t^B, evolves as follows

$$dq_t^B = q_t^B \mu_t^B dt + q_t^B \sigma_t^B dZ_t.$$

Under this conjecture, the HJB equation is

$$\rho v_t dt = \max_{dy_t \in \mathbb{R}} \left\{ \left(1 - q_t^B\right) \mathbb{I}_{\{dy_t > 0\}} e_t dy_t + \left(q_t^B - 1 - \chi\right) \mathbb{I}_{\{dy_t < 0\}} e_t \left(-dy_t\right) \right\}$$
$$+ \mu_t^B q_t^B e_t + \max_{x_t \geq 0} \left\{ r_t + x_t \left(R_t - \delta - r_t\right) - x_t \gamma_t^B \sigma \right\} q_t^B e_t - \iota q_t^B e_t,$$

where $\gamma_t^B = -\sigma_t^B$. Dividing both sides by $q_t^B e_t$, we eliminate e_t in the HJB equation,

$$\rho = \max_{dy_t \in \mathbb{R}} \left\{ \frac{\left(1 - q_t^B\right)}{q_t^B} \mathbb{I}_{\{dy_t > 0\}} dy_t + \frac{\left(q_t^B - 1 - \chi\right)}{q_t^B} \mathbb{I}_{\{dy_t < 0\}} \left(-dy_t\right) \right\}$$
$$+ \mu_t^B + \max_{x_t \geq 0} \left\{ r_t + x_t \left(R_t - \delta - r_t\right) - x_t \gamma_t^B \sigma \right\} - \iota,$$

and thus, confirm the conjecture of linear value function.

q_t^B is the marginal value of equity. Paying out one dollar of dividend, the bank's shareholders receive 1, but lose q_t^B. Only when $q_t^B \leq 1$, $dy_t > 0$. When the bank issues equity, it incurs a dilution cost. From the existing shareholders' perspective, one dollar equity is sold to outside investors at price $\frac{q_t^B}{1+\chi}$. To raise $\left(-dy_t\right) e_t$ that is worth $q_t^B \left(-dy_t\right) e_t$, the bank must issue $\frac{(1+\chi)(-dy_t)e_t}{q_t^B}$ shares, and thus, the existing shareholders give up total value of $q_t^B \frac{(1+\chi)(-dy_t)e_t}{q_t^B} = (1+\chi)\left(-dy_t\right) e_t$. Therefore, the bank raises equity only if $q_t^B \geq 1 + \chi$. Finally, the indifference condition for x_t is $R_t - \delta - r_t = \gamma_t \sigma$. If $R_t - \delta - r_t < \gamma_t \sigma$, x_t is set to zero; if $R_t - \delta - r_t > \gamma_t \sigma$, x_t is set to infinity.

Proof of Proposition 1$'$. Proof is provided in the main text.

Proof of Corollary 1$'$. Proof is provided in the main text.

Proof of Lemma 4. First, I derive equation (11). Because individual banks share the same μ_t^e, σ_t^e, and payout/issuance rate dy_t, aggregating over banks, the law of motion of E_t is

$$dE_t = \mu_t^e E_t dt + \sigma_t^e E_t dZ_t - dy_t E_t.$$

Given the expected growth rate, $\lambda F\left(m_t\right) - \delta$, which is the investment net of expected depreciation, the aggregate capital stock, K_t, evolves as: $dK_t = \left[\lambda F\left(m_t\right) - \delta\right] K_t dt + \sigma K_t dZ_t$.

By Itô's lemma, the ratio, $\eta_t = \frac{E_t}{K_t}$, has the following law of motion:

$$d\eta_t = \frac{1}{K_t}dE_t - \frac{E_t}{K_t^2}dK_t + \frac{1}{K_t^3}\langle dK_t, dK_t \rangle - \frac{1}{K_t^2}\langle dE_t, dK_t \rangle,$$

where $\langle dX_t, dY_t \rangle$ denotes the quadratic covariation of diffusion processes X_t and Y_t, so we have $\langle dK_t, dK_t \rangle = \sigma^2 K_t^2 dt$, and $\langle dE_t, dK_t \rangle = \sigma_t^e \sigma E_t K_t dt$. Dividing both sides by η_t, we have

$$\frac{d\eta_t}{\eta_t} = \frac{dE_t}{E_t} - \frac{dK_t}{K_t} + \sigma^2 dt - \sigma_t^e \sigma dt.$$

Substituting the law of motions of E_t and K_t into the equation above, we have Equation (11). The boundaries are given by banks' optimal payout and issuance policies in Proposition 3′.

Proof of Proposition 3. Following Brunnermeier and Sannikov (2014), I derive the stationary probability density. Probability density of η_t at time t, $p(\eta, t)$, has Kolmogorov forward equation

$$\frac{\partial}{\partial t} p(\eta, t) = -\frac{\partial}{\partial \eta}\left(\eta \mu^\eta(\eta) p(\eta, t)\right) + \frac{1}{2}\frac{\partial^2}{\partial \eta^2}\left(\eta^2 \sigma^\eta(\eta)^2 p(\eta, t)\right).$$

Note that in a Markov equilibrium, μ_t^η and σ_t^η are functions of η_t. A stationary density is a solution to the forward equation that does not vary with time (i.e. $\frac{\partial}{\partial t}p(\eta, t) = 0$). So I suppress the time variable, and denote stationary density as $p(\eta)$. Integrating the forward equation over η, $p(\eta)$ solves the following first-order ordinary differential equation within the two reflecting boundaries:

$$0 = C - \eta \mu^\eta(\eta) p(\eta) + \frac{1}{2}\frac{d}{d\eta}\left(\eta^2 \sigma^\eta(\eta)^2 p(\eta)\right), \quad \eta \in \left[\underline{\eta}, \overline{\eta}\right].$$

The integration constant C is zero because of the reflecting boundaries. The boundary condition for the equation is the requirement that probability density is integrated to one (i.e. $\int_{\underline{\eta}}^{\overline{\eta}} p(\eta)\, d\eta = 1$).

Next, I solve the expected time to reach from $\underline{\eta}$. Define $f_{\eta_0}(\eta)$ the expected time it takes to reach η_0 starting from $\eta \leq \eta_0$. Define $g(\eta_0) = f_{\eta_0}\left(\underline{\eta}\right)$ the expected time to reach η_0 from $\underline{\eta}$. One has to reach $\eta \in \left(\underline{\eta}, \eta_0\right)$ first and then reach η_0 from η. Therefore, $g(\eta) + f_{\eta_0}(\eta) = g(\eta_0)$. Since $g(\eta_0)$ is constant, we differentiate both sides to have $g'(\eta) = -f'_{\eta_0}(\eta)$ and $g''(\eta) = -f''_{\eta_0}(\eta)$.

From η_t, the expected time to reach η_0, denoted by $f_{\eta_0}(\eta_t)$, is decomposed into $s - t$, and $E_t\left[f_{\eta_0}(\eta_s)\right]$, i.e., the expected time to reach η_0 from η_s ($s \geq t$) after $s - t$ has passed. We have $f_{\eta_0}(\eta_t)$ equal to $E_t\left[f_{\eta_0}(\eta_s)\right] + s - t$. Therefore, $t + f_{\eta_0}(\eta_t)$ is a martingale, so f_{η_0} satisfies the ordinary differential equation: $1 + f'_{\eta_0}(\eta)\mu^\eta(\eta) + \frac{\sigma^\eta(\eta)^2}{2}f''_{\eta_0}(\eta) = 0$. Therefore, $g(\eta)$ must satisfy

$$1 - g'(\eta) \mu^\eta(\eta) - \frac{\sigma^\eta(\eta)^2}{2} g''(\eta) = 0.$$

It takes no time to reach $\underline{\eta}$, so $g(\underline{\eta}) = 0$. Moreover, since $\underline{\eta}$ is a reflecting boundary, $g'(\underline{\eta}) = 0$.

Proof of Proposition 4. The Markov equilibrium is time-homogeneous, so I suppress time subscripts. In the main text, I show how to uniquely solve x, m, and r as functions of $(q^K(\eta), q^B(\eta))$ and their derivatives. Once we know these variables, we solve the dynamics of E_t:

$$\mu^e = r + x^B (R - \delta - r), \text{ and } \sigma^e = x^B \sigma,$$

and the economic growth rate, $\lambda F(m_t) - \delta$. So, the we have the drift and diffusion of η_t:

$$\mu^\eta = \mu^e - [\lambda F(m_t) - \delta] - \sigma^e \sigma + \sigma^2, \quad \text{and} \quad \sigma^\eta = (x - 1) \sigma.$$

Next, we can use bankers' HJB equation and the capital pricing formula (i.e. Equations (13) and (17)) to form a system of differential equations for $(q^K(\eta), q^B(\eta))$, i.e., a mapping from $\left(\eta, q^B, q^K, \frac{dq^B}{d\eta}, \frac{dq^K}{d\eta}\right)$ to $\left(\frac{d^2q^B}{d\eta^2}, \frac{d^2q^K}{d\eta^2}\right)$. In stead of the first derivatives, we can work with elasticities of (q^B, q^K), $\epsilon^X = \frac{dq^X/q^X}{d\eta/\eta}$, $X \in \{B, K\}$ to simplify the expressions. Using Itô's lemma, we know

$$\mu^X = \epsilon^X \mu^\eta + \frac{1}{2q^X} (\sigma^\eta \eta)^2 \frac{d^2q^X}{d\eta^2}, \text{ i.e., } \frac{d^2q^X}{d\eta^2} = 2q^X \frac{\left(\mu^X - \epsilon^X \mu^\eta\right)}{(\sigma^\eta \eta)^2}, X \in \{B, K\}.$$

To calculate μ^K and μ^K, we use banks' HJB equation (Equation (13)):

$$\mu^B = \rho + \iota - r,$$

and the capital pricing formula (Equation (17)),

$$\mu^K = \rho + \delta - \sigma^K \sigma - \frac{\alpha}{q^K} + (\rho - r) m - \lambda \left[F(m) - \frac{m}{q^K}\right], \text{ where } \sigma^K = \epsilon^K \sigma^\eta.$$

Because $F(\cdot)$ is concave, Equation (16)) has a unique solution of x. Following from it, we solve m_t and r_t uniquely as shown in the main text. Thus, the mapping from $\left(\eta, q^B, q^K, \frac{dq^B}{d\eta}, \frac{dq^K}{d\eta}\right)$ to $\left(\frac{d^2q^B}{d\eta^2}, \frac{d^2q^K}{d\eta^2}\right)$ is unique. Given the boundary conditions, the system of differential equations uniquely pins down a solution $(q^B(\eta), q^K(\eta))$ under proper parameter range that guarantees existence, so we have a unique Markov equilibrium with state variable η_t. To solve the problem with government debt, we only need to change firms' deposit demand as shown in the main text.

Appendix II   Calibration

Table A.1: Calibration.

This table summarizes the parameter values of the solution and corresponding model and data moments (including the sources and sample size) used in calibration. Model moments are calculated using the stationary distribution of the model solution.

	Parameters		Model Moments		Data	Source
(1)	ρ	4.00%	Interest rate $\mathbb{E}\left[r_t\right]$	3.78%	3.77%	Average of MZM and T-bill rate FRED (1974Q1-2015Q4)
(2)	Inv. tech.	$F\left(i\right)=\omega_0 i^{\omega_1}$				
	$\omega0$	0.801	Expected capital growth	0.74%	0.74%*	Corrado and Hulten (2010)
	ω_1	0.99	Cash to net assets $\mathbb{E}\left[m_t\right]$	29.3%	29.2%	Compustat (1971-2015)**
(3)	λ	1/5	Expected liquidity premium $\mathbb{E}\left[\rho-r_t\right]$	24.59bps	23.65bps	GC Repo/T-bill spread Nagel (2016)
(5)	χ	1	Liquidity premium s.d. $\mathbf{std}\left[\rho-r_t\right]$	12.35bps	18.19bps	GC Repo/T-bill spread Nagel (2016)
(6)	α	0.1	Firms' equity P/E ratio	25.6	24.9	S&P 500 (Jan1991-Dec2015)
(7)	ι	2.55%	Operation cost / bank total income $\mathbb{E}\left[\frac{\iota e_t}{(R_t-\delta-r_t)x_t e_t}\right]$	90.3%	91.4%	Opex / (Opex + Net Income) Call Reports (1976Q1-2015Q4)
(8)	δ	4.00%			3.67%	Average Loan delinquency rate FRED (1985Q1-2015Q4)
(9)	σ	2.00%			1.62%	Loan delinquency rate s.d. FRED (1985Q1-2015Q4)

* In reality, economic growth is driven by both cash-intensive investments and investments that can largely rely on external financing. I set ω_0 to 0.801, so the mean growth rate matches U.S. economic growth from intangible investment (Corrado and Hulten (2010)).

** Before 1971, money market funds hadn't developed, so under Regulation Q, firms' deposit holdings did not pay interests, which is different from model.

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