# Firm Quality Dynamics and the Slippery Slope of Credit Intervention *

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**Abstract**

A salient trend in crisis intervention has emerged in recent decades: Government and central banks offered funding directly to nonfinancial firms, bypassing banks and other credit intermediaries. We analyze the long-term consequences of such policies by focusing on firm quality dynamics. In a laissez-faire economy, firms with high productivity are more likely to survive crises than those with low productivity. The government funding support saves more firms but cannot be customized based on firm productivity, dampening the cleansing effect of crises. The policy distortion is self-perpetuating: A downward bias in firm quality distribution necessitates interventions of greater scale in future crises. Our mechanism is quantitatively important: we show that if policy makers ignore such distortionary effects on firm quality dynamics, the resultant credit intervention would almost double the optimal amount.

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# 1 Introduction

Since the Global Financial Crisis (GFC), credit intervention has grown in size and become more direct. The programs during the GFC injected liquidity through the banking sector, with only a few exceptions of direct funding for nonfinancial firms, such as the bond purchase programs at the Bank of Japan and European Central Bank. During the Covid-19 crisis, not only the GFC-era programs were promptly reinstated, but central banks and governments across the world initiated new programs that directly provided liquidity support to nonfinancial firms, such as the Primary and Secondary Market Corporate Credit Facilities (PMCCF and SMCCF), Main Street Lending Program (MSLP), and Paycheck Protection Program (PPP) in the U.S.

This observation underscores a trend where credit intervention has been ambitiously designed under the mantra of doing "whatever it takes" to salvage the economy. In light of this trend, it's important to consider: What could be the long-term effects of credit intervention, and in particular, what causes credit intervention to grow in scale from one crisis to the next?

We answer this question from the perspective of firm quality dynamics. In our model, firms accumulate capital and produce, but they differ in productivity. High-quality firms have larger financing capacity because, relative to low-quality firms, they are less likely to hold up creditors through strategic default and prefer continuing operations. Therefore, high-quality firms have better access to liquidity and thus are more likely to survive in crises.[1] Crises exhibit a cleansing effect: The economy emerges from crises with a higher fraction of firms being high-quality.

Funding support from the government helps firms survive crises but dampens the cleansing effect because, unlike private-sector creditors, the government is unable to differentiate firms by their productivity. Therefore, credit intervention alleviates the decline of aggregate output but reduces the average firm productivity. As a result, a slippery slope of intervention emerges: The economy enters into the next crisis with lower productivity and thus requires a greater scale of intervention to sustain the output. In our calibrated model, increasing the scale of intervention in the current crisis by one dollar per unit of firm capital leads to an increase of 4 cents per unit of capital in the next intervention should another crisis happen in ten years. Given that capital stock grows over time, the inter-crisis pass-through rate in the total dollar amount is even larger.

A key feature of our model is the trade-off between quantity and quality. Optimal intervention

[1] A larger financing capacity leads to a higher survival probability but does not imply that high-quality firms are less financially constrained, because the tightness of financial constraint is measured by the gap between funding and the targeted level of spending. High-quality firms may also have a higher spending target as we shall explain later.

in crises strikes a balance between distorting firm quality dynamics and preserving overall production capacity. It is important for policymakers to recognize our mechanism when deciding on the intervention scale. We show that ignoring the distortionary effects on firm quality dynamics results in a more aggressive intervention that almost doubles the welfare-maximizing amount.

In the following, we summarize the key modeling ingredients and provide more details on our main results. We follow the continuous-time formulation of the multi-sector models (Eberly and Wang, 2008) but simplify households' preferences to be risk-neutral. There are two types of firms that produce generic goods for consumption and investment using their productive capital with constant return-to-scale technology. Type-$H$ firms have a higher productivity than type-$L$ firms. In normal times, firms are not financially constrained and invest to grow their capital. The forward-looking valuation of capital (i.e., Tobin's q) is the present value of production flows and drives the optimal investment (Hayashi, 1982; Abel and Eberly, 1994; Brunnermeier and Sannikov, 2014).

The arrival of a crisis follows a Poisson process (Gourio, 2012; Wachter, 2013). We model a liquidity crisis in the spirit of Holmström and Tirole (1998). Firms differ in their crisis exposure: each firm draws a baseline level of survival probability from a common distribution unrelated to its type. Firms can raise debt financing to increase survival probability. Type-$H$ firms have a higher targeted level of spending on survival as their capital is more valuable than that of type-$L$ firms. The first form of cleansing effect emerges: Type-$H$ firms want to spend more, which may translate into a higher survival probability. Our emphasis shall be on the second form of cleansing effect: Type-$H$ firms can borrow more than type-$L$ firms.[2] It is the interference with this force that causes policy intervention to distort firm quality dynamics and exhibit intertemporal dependence.

Firms face a debt limit imposed by private-sector creditors because they may hold up creditors through strategic default (e.g., Bolton and Scharfstein, 1990; Hart and Moore, 1998). In strategic default, firm owners lose capital to the creditors but can extract value from the creditors through renegotiation.[3] Therefore, type-$L$ firms have a stronger incentive to strategically default than type-$H$ firms because, due to lower productivity, their capital is less valuable. The fact that type-$L$ firms face a tighter debt limit than type-$H$ firms gives rise to the second form of cleansing effect.

[2]The cleansing effects of crises in our model are distinct from those via creative destruction or cyclical reallocation(e.g., Eisfeldt and Rampini, 2006; Kogan et al., 2017; Acemoglu et al., 2018) and the efficiency gain is from creditors' disciplining firms against over-spending rather than weeding out unproductive firms that crowd out productive peers in product or input markets (e.g., Caballero and Hammour, 1994; Acharya et al., 2021).

[3]Firm owners' bargaining power may originate from their threat to divert resources, engage in wasteful spending, or withhold human capital (Aghion and Bolton, 1992; Hart and Moore, 1994).

This cleansing effect arises from the link between firm type and financing capacity. This link is quite general and can be modeled differently, for example, through a collateral constraint, in which case type-$H$ firms pledge their more valuable capital as collateral and can raise more funds.[4] Our modeling choice is motivated by the empirical literature on credit intervention.[5]

The government extends credit to firms in order to help more firms survive the crisis, effectively acting as a financial intermediary (Lucas, 2016). It finances lending with taxes on households and transfers the repayments in lump sums, in line with the models of credit policy and unconventional monetary policy (e.g., Gertler and Karadi, 2011a; Gertler et al., 2012; Araújo et al., 2015; Del Negro et al., 2017). When setting interest rates, the government follows private-sector creditors as in practice.[6] However, when setting the size of credit support, the government imposes a uniform limit to firms of both types: A firm can borrow up to a multiple of its capital stock. This reflects the policy design in reality: Credit limit is set proportional to accounting measures of operation scale, which map to the capital units in our model, such as the programs during the Covid-19 crisis.[7]

Firms can choose any amount of borrowing from the credit facilities within the limit set by the government. The impact of credit intervention cannot be solely judged by the take-up. In our model, credit intervention enlarges firms' financing capacity both directly through the liquidity available and indirectly by crowding in private-sector funding. By improving the survival probability, credit intervention also affects the pricing of credit by private-sector creditors. Any change in the scale of credit intervention results in a new equilibrium of financing capacity and costs.

Credit intervention benefits type-$L$ firms more than type-$H$ firms, dampening the cleansing effect of crises. Intuitively, since type-$L$ firms obtain less funds from private-sector creditors, the marginal impact of more funds from the government is greater for type-$L$ firms. While private-sector investors properly account for type-$H$ and $L$ firms' difference in incentive to hold up creditors, the government does not, and therefore, its funding support generates distortionary effects.

[4]Debt capacity depends on collateral value under limited commitment (Kehoe and Levine, 1993; Kiyotaki and Moore, 1997; Geanakoplos, 2010; Rampini and Viswanathan, 2010; Li, Whited, and Wu, 2016).

[5]The recent evidence documented in the literature suggests a significant conflict of interests between borrowers and lenders in crises (Hanson et al., 2020; Lynch, 2021; Griffin et al., 2023).

[6]For publicly traded bonds, the government can rely on market prices, e.g., PMCCF and SMCCF in the U.S., and if a firm's debt is not publicly traded, the government can lend alongside informed banks and rely on banks to screen out and exclude firms that are riskier for a given level of interest rate, e.g., MSLP during the Covid-19 pandemic.

[7]MSLP sets a limit to six times the borrower's EBITDA, a measure of operating income rather than profitability per unit of resources deployed. The limit in PPP is tied to payroll rather than labor productivity. PMCCF and SMCCF impose limits tied to an issuer's existing debts (liability size rather than productivity). In addition to the uniform credit limit, if the government forgoes differentiation on interest rates, our mechanism is amplified (Appendix B.3).

There has been enormous attention on the lack of firm differentiation in credit facilities (English and Liang, 2020). There are several explanations. The government lacks information on firm productivity.[8] Political considerations may go against treating firms differently in crises. And, customizing credit support for individual firms is infeasible when speedy implementation is required.

In summary, the productivity difference between type-$H$ and $L$ firms translates into a difference in private-sector financing capacity. A cleansing effect of crises emerges from the fact that type-$H$ firms have a greater financing capacity and thus a higher survival probability. Credit intervention dampens such cleansing effect. Since both types of firms can obtain funding from the government, our analysis focuses on policy distortions among firms that can access the facility but have different levels of productivity. In practice, credit facilities may exclude certain firms, but the criterion is rarely productivity-based and thus unlikely to help avoid distortions on firm quality distribution.[9]

In our model, the government faces a trade-off between quantity and quality in line with the empirical evidence.[10] Consider increasing credit support from zero to 30% of GDP.[11] In our calibrated model, the policy saves 8% of production capacity (i.e., capital units) but reduces the quality improvement (i.e., the increase in the fraction of firms being type-$H$) from about 9% to 4%. Optimal intervention strikes a balance between allowing more firms to survive and distorting the quality distribution. Reducing funding support strengthens the cleansing effect, so the economy has higher productivity post-crisis but has to climb out of a deeper decline in total output. If the government supplies more funding, the output drop is contained at the expense of lowering productivity.

Accordingly, the welfare is a bivariate function of the state variables, the total number of firms and fraction of firms being type-$H$. When solving the constrained-efficient scale of intervention, we first consider a policy maker that ignores the distortionary impact on quality dynamics and is only aware of the positive impact on capital quantity. Here, the trade-off is standard—investing

[8] Firms differ in both type (productivity) and crisis exposure (baseline survival probability), so the government cannot infer the type from credit pricing in the market and thus cannot properly account for firms' incentive to hold up creditors. Credit risk and productivity are correlated but not perfectly aligned in our model and data (Appendix C.1).

[9] For example, PMCCF and SMCCF introduced in the U.S. during the Covid-19 pandemic excluded firms with high credit risk. In Appendix C, we show that credit risk and productivity are far from being perfectly correlated.

[10] Intervention was effective in preserving production capacity during the Covid-19 crisis (Bartik et al., 2020; Bartlett and Morse, 2020; Hubbard and Strain, 2020; Denes et al., 2021; Kawaguchi et al., 2021). On the cleansing effect of the Covid-19 crisis, Muzi et al. (2023) find less productive firms were more likely to cease operations, and Bruhn et al. (2023) find economic activity was reallocated toward more productive firms beyond what is implied by cyclical variation. These two papers document that intervention dampens the cleansing effect. Moreover, Dörr et al. (2022) find that credit support disproportionately benefited firms that were financially vulnerable pre-Covid 19.

[11] This is close in magnitude to the size of credit support in the U.S. during the Covid-19 crisis, including various credit programs such as PPP and MSLP. See Section 3 for more discussions on the magnitude of crisis interventions.

goods in firm survival vs. consuming goods—and the resultant intervention is almost twice the size of optimal intervention that properly accounts for the impact on both state variables, quantity and quality. Therefore, ignoring the impact on quality dynamics leads to excessive intervention. This exercise shows that recognizing the mechanism in this paper is quantitatively important.

Another perspective on welfare analysis is under- vs. over-spending under credit intervention. For both type-$H$ and $L$ firms, credit intervention enlarges the financing capacity of firms that under-spend due to a large crisis exposure and induces over-spending among firms that are relatively less liquidity-constrained and over-borrow. Over-spending on survival happens because the option to hold up creditors through strategic default is in-the-money only if the firm survives. Therefore, under a sufficiently large scale of intervention, over-spending is guaranteed to happen, and it can happen among both type-$H$ and $L$ firms, though more prominent among type-$L$ firms.

Firms that over-spend on survival engage in a negative NPV transaction but are not zombies. In crises, a firm has temporary liquidity needs; after surviving the crisis, it recovers, producing goods and making investments guided by Tobin's q. Zombies are firms that are permanently impaired and continue operating only by relying on external financial resources (e.g., Caballero, Hoshi, and Kashyap, 2008; Acharya, Eisert, Eufinger, and Hirsch, 2019; Acharya, Lenzu, and Wang, 2021). In Appendix B.4, we discuss an alternative setup where the presence of zombie firms further amplifies the distortionary effects of credit intervention and make the effects more persistent.

In spite of the temporary nature of firms' liquidity problem, policy distortions persist over time. In our calibrated model, increasing credit support from zero to 30% of GDP reduces the fraction of firms being type-$H$ by 4 percentage points even ten years after the crisis. When a subsequent crisis arrives, the economy requires an even greater scale of intervention because, a lower average firm productivity translates into a smaller private-sector financing capacity in aggregate. Therefore, a slippery slope of intervention emerges in our model: Credit intervention in the current crisis begets interventions of greater scales in future crises. This slippery slope is robust even when we shut down agents' expectations of ever-growing intervention. The main takeaway from our paper is that credit intervention faces a quantity-quality trade-off, and any one-time deviation towards more aggressive intervention generates a ripple effect that permeates indefinitely into the future.

**Literature.** The role of governments, especially, the central banks, as lenders of last resort constantly evolves throughout the history in response to crises, political struggles, and technological innovations (Goodhart, 1998; Calomiris, Flandreau, and Laeven, 2016). Direct lending to nonfinancial firms is a meaningful addition to the policy toolbox. During a credit market freeze (Stiglitz and Weiss, 1981; De Meza and Webb, 1987), the government can step in, effectively functioning as a financial intermediary (Bebchuk and Goldstein, 2011; Lucas, 2016).[12] The Covid-19 crisis normalized the use of direct liquidity support to nonfinancial firms and will have a long-lasting effect on firms' expectations and their investment and financing decisions (Elenev et al., 2020).

The models of unconventional monetary policy assume an exogenous dead-weight loss of direct lending (Gertler and Kiyotaki, 2010; Cúrdia and Woodford, 2011; Gertler and Karadi, 2011a; Gertler, Kiyotaki, and Queralto, 2012; Araújo, Schommer, and Woodford, 2015; Del Negro, Eggertsson, Ferrero, and Kiyotaki, 2017). We unpack the black box of costs of government lending to nonfinancial firms, or asset purchases in general, and emphasize the endogenous evolution of firm quality distribution and a novel dynamic mechanism that leads to a slippery slope of intervention.[13]

Broadly, our paper contributes to the literature on the costs of crisis intervention, such as risk cost (Lucas, 2012), tax distortions as a form of financing costs (Hanson, Scharfstein, and Sunderam, 2018), feedback loop between sovereign and private-sector risk (Acharya et al., 2014; Brunnermeier et al., 2016), distortions on bank capital allocation (Antill and Clayton, 2021), and debt overhang and bankruptcy costs (Balloch et al., 2020; Brunnermeier and Krishnamurthy, 2020; Crouzet and Tourre, 2020; Greenwood et al., 2020; Wang et al., 2020).[14]

In our model, crises exhibit cleansing effects that emerge from type-$H$ and $L$ firms' differences in debt capacity, which is, in turn, due to creditors imposing discipline against over-spending. By dampening the cleansing effect, government intervention weakens such discipline. The efficiency gain from the crisis cleansing effect in our model is different from that in the existing models of cleansing effects that emphasize unproductive firms crowding out productive firms in output (product) or input (factor) markets and weeding out unproductive firms improves efficiency (e.g., Caballero and Hammour, 1994). The mechanism in our model is solely built on the financial aspects of crises.[15] Incorporating type-$L$ firms crowding out type-$H$ firms in product or factor

[12]Bassetto and Cui (2020) analyze tax/subsidy as an alternative to credit policy in addressing financial frictions.

[13]Beyond our emphasis of endogenous quality, our paper focuses on the intensive margin of policy intervention—among firms that receive credit support, the costs of capital of firms with different productivities are homogenized—while other studies emphasize the distortions from the extensive margin, i.e., a subset of firms receive a disproportionately large amount of credit support (Kurtzman and Zeke, 2020; Papoutsi, Piazzesi, and Schneider, 2021).

[14]The recent contributions on the benefits of credit-market intervention focus on the positive externalities that cannot be internalized by private lenders (e.g., Bebchuk and Goldstein, 2011; Philippon and Schnabl, 2013; Liu, 2016; Giannetti and Saidi, 2019; Hanson, Stein, Sunderman, and Zwick, 2020).

[15]The cleansing effects of crises in our model are also distinct from those via creative destruction or cyclical reallocation(e.g., Eisfeldt and Rampini, 2006; Kogan et al., 2017; Acemoglu et al., 2018).

markets amplifies the inefficiency of credit intervention, dampening the cleansing effect of crises.

Firm quality distribution evolves endogenously in our model under a fixed information structure. Our paper focuses on the interaction between policy intervention and firm quality and differs from studies on endogenous asset quality that emphasize an evolving information structure and agents' incentive to improve asset quality, motivated by the GFC (Eisfeldt, 2004; Chari, Shourideh, and Zetlin-Jones, 2014; Chemla and Hennessy, 2014; Kurlat, 2013; Gorton and Ordoñez, 2014; Bigio, 2015; Zryumov, 2015; Bolton, Santos, and Scheinkman, 2016; Moreira and Savov, 2017; Caramp, 2017; Hu, 2017; Vanasco, 2017; Fukui, 2018; Neuhann, 2018; Asriyan, Fuchs, and Green, 2019; Daley, Green, and Vanasco, 2020; Lee and Neuhann, 2021; Farboodi and Kondor, 2021).[16]

Our model features misallocation. Unlike the literature that studies factor allocation (Ramey and Shapiro, 1998; Eisfeldt and Rampini, 2006, 2008; Jovanovic and Rousseau, 2008), our focus is on liquidity allocation between firms and households and between high- and low-quality firms. Intervention causes misallocation among firms but improves efficiency by channeling liquidity from households to firms. We analyze the trade-off, contributing to the literature on financial frictions and misallocation (Banerjee and Moll, 2010; Gilchrist, Sim, and Zakrajšek, 2013; Midrigan and Xu, 2014; Moll, 2014; Fuchs, Green, and Papanikolaou, 2016; Dou, Ji, Tian, and Wang, 2020; David and Zeke, 2021). As is David, Schmid, and Zeke (2018), there is a connection between firm productivity and risk exposure: Type-$H$ firms, by having larger financing capacity, are less affected by crises. Intervention distorts the firm quality dynamics by changing firms' crisis exposure.

# 2 The Model

## 2.1 Preferences and Technology

Consider a continuous-time economy with a unit measure of representative agents ("households") and a government. Households have risk-neutral utility with time discount rate $r$:

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-rt} dc_t\right], \tag{1}$$

[16] Policy makers may actively alter the information structure, which in turn affects the optimal intervention (Goldstein and Sapra, 2014; Bouvard, Chaigneau, and Motta, 2015; Shapiro and Skeie, 2015; Williams, 2015; Faria-e-Castro, Martinez, and Philippon, 2016; Goldstein and Leitner, 2018).

where $c_t$ is the cumulative consumption. Households can own and trade equity and debt of firms. Firms maximize shareholders' value by managing capital to produce non-durable numeraire goods.

There are two types of firms. Type-$H$ firms' capital produces $A^H$ units of goods per unit of time. The productivity of type-$L$ capital is $A^L$ ($A^H > A^L$). Capital depreciates at rate, $\delta$. We use superscripts for type and subscripts for time. Given the aggregate capital stocks, $K_t^H$ and $K_t^L$, the total output of numeraire goods over $dt$ is

$$Y_t \equiv A^H K_t^H + A^L K_t^L = \left(A^H \omega_t + A^L (1-\omega_t)\right) K_t \,, \tag{2}$$

where $\omega_t$, the fraction of type-$H$ capital, represents the firm quality distribution

$$\omega_t \equiv \frac{K_t^H}{K_t^H + K_t^L} \,, \tag{3}$$

and $K_t$ is the total units of capital

$$K_t \equiv K_t^H + K_t^L \,. \tag{4}$$

The output in the economy depends on both the capital quality, $\omega_t$, and capital quantity, $K_t$.

Firms can grow capital through investment. Let $q_t^j$, $j \in \{H, L\}$, denote the endogenous value of capital that will be solved in equilibrium. It plays an important role in our analysis, as it incorporates the expectation of future growth path and disruptions in crises. Given the time-$t$ value of capital, $q_t^j$, a type-$j$ firm chooses the investment rate (or capital growth rate), $\iota_t^j$:

$$\max_{\iota_t^j} q_t^j \iota_t^j k_t - \Phi(\iota_t^j, k_t) \,, \tag{5}$$

where we adopt the cost function from the literature on Q-theory of investment (Hayashi, 1982):

$$\Phi(\iota_t^j, k_t) = \underbrace{\left(\iota_t^j + \frac{\theta}{2} {\iota_t^j}^2\right)}_{\phi(\iota_t^j)} k_t \tag{6}$$

Under this functional form, we obtain the classic Q-theory formula of optimal investment:

$$\iota_t^j = \frac{q_t^j - 1}{\theta} \,. \tag{7}$$

The arrival of crises follows a Poisson process, $N_t$, with intensity $\lambda$ (e.g., Gourio, 2012; Wachter, 2013). We model a liquidity crisis in the spirit of Holmström and Tirole (1998), where firms need to inject additional resources to preserve their production capacity. In a crisis (i.e., $dN_t = 1$), firms raise money for survival. Let $x_t \geq 0$ denote the amount of financing per unit of capital a firm obtains and spends on surviving the crisis. The firm then faces a survival probability $F(x_t + \zeta)$, where $\zeta$ is drawn from a common cumulative distribution function $H(\zeta)$ independently across firms and is realized before the firm makes its financing decision $x_t$. We assume that $F(\cdot)$ is strictly increasing and concave with $F(0) = 0$, so a higher $\zeta$ means a higher chance of survival (since $F'(\cdot) > 0$) and a lower marginal value of liquidity (since $F''(\cdot) < 0$). The random variable $\zeta$ captures different factors that affect the severity of a firm's liquidity crisis. For example, a low $\zeta$ reflects a severe mismatch between cash inflows and outflows, such as customers delaying payment and suppliers suspending trade credit. A firm's decision to raise external funds depends on its type $j \in \{H, L\}$ (through the capital value $q_t^j$) and the severity of its liquidity crisis given by $\zeta$, so we denote the optimal amount of financing per unit of capital as $x_t^j(\zeta)$, $j \in \{H, L\}$.

Incorporating both normal-time growth through investment and crisis-time exit with probability $1 - F\left(x_t^j(\zeta) + \zeta\right)$, aggregate type-$j$ capital stock has the following law of motion:

$$dK_t^H = K_t^H(\iota_t^H - \delta)dt - K_t^H \left[\int_\zeta \left(1 - F\left(x_t^H(\zeta) + \zeta\right)\right) dH(\zeta)\right] dN_t, \tag{8}$$

$$dK_t^L = K_t^L(\iota_t^L - \delta)dt - K_t^L \left[\int_\zeta \left(1 - F\left(x_t^L(\zeta) + \zeta\right)\right) dH(\zeta)\right] dN_t + \eta K_t dt. \tag{9}$$

We will show that in equilibrium, $q_t^H > q_t^L$, so type-$H$ firms grow faster in normal times ($\iota_t^H > \iota_t^L$). Moreover, we will show that type-$H$ firms have higher survival rates in crises. To maintain the stationarity of quality distribution, we introduce exogenous birth $\eta K_t dt$ for L-type firms.

## 2.2 Financial Frictions and Government Intervention in Crises

First, we establish an efficiency benchmark. Consider a social planner's choice of $x_t^j(\zeta)$, the spending on survival by a firm of type-$j$, $j \in \{H, L\}$ with a realized $\zeta$. When spending $x$ per unit of capital, probability of preserving the capital is $F(x + \zeta)$ and the expected payoff is $F(x + \zeta)q_t^j$:

$$\max_x F(x + \zeta)q_t^j - x\,. \tag{10}$$

In equation (10), we consider a liquidity crisis in which firms spend real resources for survival, following Holmström and Tirole (1998). The planner balances the social benefit from increasing the firm's survival probability and the social cost of using up real resources (the generic goods).[17] Under a strictly concave $F(\cdot)$, the optimal amount of spending is given by first-order condition:

$$F'(x_t^{*j}(\zeta) + \zeta)q_t^j = 1 \tag{11}$$

for $x_t^{*j}(\zeta) > 0$, and $x_t^{*j}(\zeta) = 0$ if $F'(x_t^{*j}(\zeta) + \zeta)q_t^j = F'(\zeta)q_t^j < 1$ as the amount of financing cannot be negative. We use $x_t^{*j}(\zeta)$ to denote the socially optimal (first-best) level.

Next, we model debt financing offered by private-sector creditors and show that, without frictions, the credit market equilibrium generates the social optimum. The creditors charge an interest rate $r_t^j(\zeta, x)$ on a type-$j$ firm with realized $\zeta$ and debt level $x$ per unit of capital. The firm chooses the amount of debt financing to maximize the expected value:

$$x_t^j(\zeta) = \arg \max_{0 \le x \le \bar{d} + \bar{g}} F(x + \zeta) \left[ q_t^j - (1 + r_t^j(\zeta, x))x \right] . \tag{12}$$

The firm survives with probability $F(x+\zeta)$, and after paying the principal and interest to creditors, the firm owners' value is $q_t^j - (1 + r_t^j(\zeta, x))x$. The private-sector credit supply is constrained at $\bar{d}$. Government intervention, $\bar{g}$, relaxes the constraint. Note that what distinguishes crises and normal times is the limited private-sector credit in crises. In normal times, firms' investment is unconstrained, given by (7). Therefore, the tightening of credit conditions in crises reminisces the concept a financial shock (Jermann and Quadrini, 2012). Credit freeze can be attributed to lenders' balance-sheet impairment, elevated uncertainty, or informational frictions.[18]

[17] During the Covid-19 pandemic, firms adapted products to survive. From the restaurant industry to software engineering, businesses altered products and production processes, and doing so entails real resources spent on human capital (e.g., redeploying and training workers to produce new products), and materials and real estate (e.g., e-commerce platforms building new warehouses to address heightened logistics demand). Spending such real resources is important for firms' survival. One may argue that certain liquidity crises do not entail spending real resources and survival only requires bridge financing. For example, firms experiencing delay in customer payments need external financing to bridge through liquidity shortage. In Appendix D.5, we consider an extended model to allow for this possibility.

[18] Credit freeze happens for various reasons, such as lenders' lack of capital (Bernanke and Lown, 1991), information decay in booms (Gorton and Ordoñez, 2014; Asriyan, Laeven, and Martin, 2018), foreigners' withdrawal (Van Nieuwerburgh and Veldkamp, 2009; Koijen, Koulischer, Nguyen, and Yogo, 2020), and ambiguity in risk evaluation Boyarchenko (2012); Caballero and Simsek (2013); Drechsler (2013). A market crash happened during the global financial crisis (Acharya, Schnabl, and Suarez, 2013; Brunnermeier, 2009; Gorton, Laarits, and Metrick, 2017; Kacperczyk and Schnabl, 2010; Krishnamurthy, 2010). Credit markets were under tremendous stress during the Covid-19 pandemic (Falato, Goldstein, and Hortaçsu, 2020; Haddad, Moreira, and Muir, 2020; Halling, Yu, and Zechner, 2020;

The interest rate, $r_t^j(\zeta, x)$, is set by the creditors' break-even condition:

$$F(x+\zeta)(1+r_t^j(\zeta,x))x = x\,. \tag{13}$$

When lending to firms, the government also charges this interest rate, taking advantage of the market signals. There are different types of policy design that allow the government to rely on market signals. Examples include PMCCF and SMCCF in the U.S. during the Covid-19 pandemic.[19]

Using (13), we substitute out $r_t^j(\zeta, x)$ in the firm value in (12) and obtain a simplified objective function $F(x+\zeta)q_t^j - x$, which is exactly the planner's objective function given by (10). Therefore, without the funding constraints (i.e., with $\bar{d}$ set to be infinite), the credit market equilibrium generates the socially optimal (first-best) level of spending on survival, $x_t^{*j}(\zeta)$ given by (11). To simplify the discussion below, we introduce a notation, $\bar{\zeta}_t^j$, defined through

$$F'(\bar{\zeta}_t^j)q_t^j = 1\,. \tag{14}$$

Under $q_t^H > q_t^L$ and a strictly concave $F(\cdot)$, we have $\bar{\zeta}_t^H > \bar{\zeta}_t^L$ from (14). Type-$H$ capital is naturally more valuable than type-$L$ capital because type-$H$ firm has a higher productivity, i.e., $A^H > A^L$. We will later formally show $q_t^H > q_t^L$ after fully solving the endogenous capital values in equilibrium. The following lemma summarizes the first-best levels of financing.

**Lemma 1 (First best)** *The first-best level of financing for a type-$j$ firm with realized $\zeta$ is given by*

$$x_t^{*j}(\zeta) = (\bar{\zeta}_t^j - \zeta)^+, \tag{15}$$

*where $j \in \{H,\, L\}$. Under $q_t^H > q_t^L$, we have $\bar{\zeta}_t^H > \bar{\zeta}_t^L$.*

A firm's optimal level of financing is the minimum of first-best level and the available funding:

$$x_t^j(\zeta) = \min\{x_t^{*j}(\zeta),\, \bar{d} + \bar{g}\}\,. \tag{16}$$

Government intervention improves efficiency by bringing $x_t^j(\zeta)$ closer to $x_t^{*j}(\zeta)$. Here the government acts as a financial intermediary in crises (Lucas, 2016). It finances lending with lump-sum

Kargar, Lester, Lindsay, Liu, Weill, and Zúñiga, 2020; Ma, Xiao, and Zeng, 2020).

[19] If a firm's debt is not publicly traded, the government can "free-ride" banks' information production in pricing or screening (for example, the government relied on banks to screen borrowers in MSLP during the Covid-19 pandemic).

taxes on deep-pocket households and transfers the repayments to households.

So far, the only source of inefficiency is the limited funding supply in the private sector, and the optimal intervention simply requires $\bar{g}$ to be sufficiently large so that $x_t^j(\zeta) = x_t^{*j}(\zeta)$. However, funding in crises is limited because of frictions not only on the credit supply side but also on the demand side. Introducing borrowers' moral hazard allows credit intervention to have potentially harmful effects: a large scale intervention can lead to over-spending on survival in crises.

We model borrowers' moral hazard in the form of strategic default. Up to this point, we have assumed that firms repay debts if they survive. However, as well-known in the corporate finance literature (Bolton and Scharfstein, 1990; Hart and Moore, 1998), there are two types of default. First, a firm defaults because it cannot survive, which happens with a probability $1-F\left(x_t^j(\zeta)+\zeta\right)$. Second, a firm survives but its owners extract value from creditors through strategic default and renegotiation. The owners may threaten to divert resources, engage in wasteful spending, or withhold human capital (Aghion and Bolton, 1992; Hart and Moore, 1994). After renegotiation, the firm owners obtain $\beta$ while the private-sector creditors seize the capital and obtain $q_t^j - \beta$ as the control right is typically transferred to creditors when default happens. The recovery value is zero for the government, which is in line with evidence on the prevalent abuse of government funding during crises (Hanson et al., 2020; Lynch, 2021; Griffin et al., 2023). Also it is rare for the government to engage in bankrupt firms' business restructuring (seizing the capital and continuing operating). For firms that strategically default, government funding is essentially a subsidy on survival.

A firm that has survived the crisis compares the value per unit of capital from strategic default, $\beta$, and the value under debt repayment, $q_t^j - \left[1+r_t^j\left(\zeta, x_t^j(\zeta)\right)\right] x_t^j(\zeta)$. Firms will strategically default if and only if it is strictly better than honoring debt repayment. Let $\underline{\zeta}_t^j$ denote the solution to the following indifference condition defined over $\zeta$,

$$q_t^j - \left[1+r_t^j\left(\underline{\zeta}_t^j, x_t^j(\underline{\zeta}_t^j)\right)\right] x_t^j(\underline{\zeta}_t^j) = \beta, \tag{17}$$

if a solution exists. Note that if $\beta$ is sufficiently high, for example, $\beta = q_t^j$, then all firms prefer strategic default over debt repayment and the indifference condition may never hold for any $\zeta$. If $\beta = 0$, then all firms repay their debts. Therefore, the equation (17) may not have a solution. The following lemma shows that when the solution of equation (17) exits, it is unique, and there are less type-$H$ firms that strategically default than type-$L$ firms. The full proof is in the appendix.[20]

[20]The intuition behind the proof is as follows. Given $q_t^j$, we compute the value under debt repayment, $q_t^j - [1+$

**Lemma 2 (Strategic default)** *A type-$j$ firm with realized $\zeta$ strategically defaults if and only if $\zeta < \underline{\zeta}_t^j$, where $\underline{\zeta}_t^j$ as a solution to (17) is unique. Furthermore, under $q_t^H > q_t^L$, we have $\underline{\zeta}_t^H < \underline{\zeta}_t^L$.*

Firms with a lower $\zeta$ have more liquidity needs and borrow more, so they tend to strategically default. Intuitively, if a firm has more debt obligations, default saves a larger repayment. Default also causes the firm owners to lose capital ownership, so when capital is more valuable ($q_t^j$ is higher), firms tend not to strategically default. Given that $q_t^L < q_t^H$ implies $\underline{\zeta}_t^L > \underline{\zeta}_t^H$, there are more type-$L$ firms that strategically default ($\zeta \in (0, \underline{\zeta}_t^L)$) than type-$H$ firms.

For firms that strategically default, their objective function is $F(x_t^j(\zeta) + \zeta)\beta$ without concern over repayment. Therefore, they simply borrow as much as possible to maximize survival probability. Private-sector creditors know $q_t^j$ (and hence the strategic-default threshold $\underline{\zeta}_t^j$), and they know firms' draw of $\zeta$, so private-sector creditors know which firm will repay debts and which will strategically default. When lending to firms that will strategically default, private-sector creditors understand that such firms will borrow as much as possible. Therefore, beyond the interest rate $r_t^j\left(\zeta, x_t^j(\zeta)\right)$, private-sector creditors also specify a debt limit, denoted by $\hat{d}_t^j(\zeta)$,

$$\hat{d}_t^j(\zeta) = F(\hat{d}_t^j(\zeta) + \zeta + \bar{g})(q_t^j - \beta)\,, \tag{18}$$

to make sure that they break even.[21] Note that the interest rate is relevant for firms that strategically default even though they do not repay their debts, because the decision to default or not is based on comparing the value in strategic default and the value after debt repayment, as shown in (17).

**Lemma 3 (Endogenous debt capacity)** *For any $\zeta$, $\hat{d}_t^j(\zeta)$ in (18) exists and is unique. It is strictly increasing and concave in $\zeta$. Furthermore, under $q_t^H > q_t^L$, we have $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$.*

We summarize how events unfold in a crisis. The competitive private-sector creditors are informed about firms' type $j$ and crisis exposure that is inversely indexed by $\zeta$. Therefore, they offer loan contracts indexed by $j$ and $\zeta$. The contractual interest rate is set by the creditors' breakeven

$r_t^j(\zeta, x_t^j(\zeta))]x_t^j(\zeta)$ for all values of $\zeta$, which we prove is a monotonic function of $\zeta$. This function incorporates the interest-setting function $r_t^j(\zeta, x)$ given by (13) and a firm's optimal borrowing $x_t^j(\zeta)$ function given by (16). The threshold $\underline{\zeta}_t^j$ is where this value under debt repayment as a function of $\zeta$ crosses $\beta$, the value under strategic default.

[21]Note that if $\hat{d}_t^j(\zeta) \geq \bar{d}$, $x_t^j(\zeta) = \bar{d} + \bar{g}$, in which case the creditors earn positive profits because, under a strictly increasing and concave $F(\cdot)$, $F(\bar{d}+\bar{g}+\zeta)(q_t^j - \beta) > \bar{d}$. The creditors may want to lend more but they are constrained by the amount of available funds, $\bar{d}$. Such funding shortage and the positive profits earned by those who can provide funding is a common feature of liquidity crises that we capture in our model.

condition (13) so that a type-$j$ firm with a realized $\zeta$ faces interest rate $r_t^j\left(\zeta, x_t^j(\zeta)\right)$. Note that the interest rate depends on the borrowing amount. For a type-$j$ firm with $\zeta$ below $\underline{\zeta}_t^j$, the creditors also imposes a borrowing limit, $\hat{d}_t^j(\zeta)$, recognizing the borrower's incentive to strategic default. As shown in (18), the borrowing limit ensures that the creditors break even when lending to these firms. For these firms, loan repayment is off-equilibrium, so one may wonder why the contracts specify interest rates according to (13). Interest rates are still important because the firm's decision to strategically default or not is made through the comparison between the value from repaying the loan and that from strategic default, as shown in the indifference condition (17). The interest rates, given by (13), imply that in the off-equilibrium scenario where these firms make repayment, the competitive creditors break even.[22] In summary, a type-$j$ firm's optimal choice of $x$ is given by

$$x_t^j(\zeta) = \underbrace{\mathbf{1}_{\zeta \geq \underline{\zeta}_t^j} \min\{(\bar{\zeta}_t^j - \zeta)^+, \bar{d} + \bar{g}\}}_{\text{no strategic default}} + \underbrace{\mathbf{1}_{\zeta < \underline{\zeta}_t^j}\left(\min\{\hat{d}_t^j(\zeta), \bar{d}\} + \bar{g}\right)}_{\text{strategic default}}, \tag{19}$$

Firms that repay debt raise funding given by (16). Firms that strategically default max out borrowing. Based on Lemma 1, 2, and 3, the following proposition summarizes firms' financing strategy.

**Proposition 1 (Equilibrium financing)** *The financing amount of a type-j firm, $j \in \{H, L\}$, with a realized $\zeta$, $x_t^j(\zeta)$, is given by (19), with $\bar{\zeta}_t^H > \bar{\zeta}_t^L$, $\underline{\zeta}_t^H < \underline{\zeta}_t^L$, and $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$ under $q_t^H > q_t^L$. It also has the following properties: (1) $x_t^j(\zeta)$ is increasing and concave in $\zeta$ for $\zeta < \underline{\zeta}_t^j$ and $\hat{d}_t^j(\zeta) \leq \bar{d}$; (2) $x_t^j(\zeta)$ is decreasing and linear in $\zeta$ for $\zeta \in [\underline{\zeta}_t^j, \bar{\zeta}_t^j]$ and $(\bar{\zeta}_t^j - \zeta)^+ \leq \bar{d} + \bar{g}$.*

For a type-$j$ firm, $j \in \{H, L\}$ with $\zeta$ above the default threshold $\underline{\zeta}_t^j$, a lower $\zeta$ (i.e., a stronger liquidity need) leads to more borrowing. For this firm, the only source of potential inefficiency is the funding limit. For a type-$j$ firm with $\zeta < \underline{\zeta}_t^j$, it faces an additional problem of endogenous debt limit. As $\hat{d}_t^j(\zeta)$ increases in $\zeta$, a firm with a stronger liquidity need (i.e., a lower $\zeta$) actually has a smaller debt capacity $\hat{d}_t^j(\zeta)$ and thus can borrow less. Moreover, given the concavity of $\hat{d}_t^j(\zeta)$ in $\zeta$ (see Lemma 3), the further $\zeta$ decreases, the faster debt capacity shrinks. This is a classic insight from the corporate finance literature: A borrower's own lack of commitment against strategic

[22] Note that for the interest rates given by equation (13), the creditors do not probability-weight scenarios of repayment conditional on survival vs. strategic default conditional on survival. The creditors know a firm's type, $j$, and $\zeta$, so they know which firms repay and which firms strategically default conditional on surviving the crisis. Therefore, the interest rates do not "price in" strategic default. For firms with $\zeta < \underline{\zeta}_t^j$, the interest rates simply specify the off-equilibrium scenario of how much to pay if they do not strategically default and behave as firms with $\zeta \geq \underline{\zeta}_t^j$. The indifference condition (17), is an incentive compatibility condition that generates the $\zeta$ threshold for strategic default.

default, which is more severe when $\zeta$ is lower and the financing need is stronger (see Lemma 2), compromises her financing capacity. The financing strategy characterized in Proposition 1 has a rich set of features that allow us to discuss different channels through which government intervention can affect firms' financing in crises and may alleviate or exacerbate inefficiencies.

**Discussion: Productivity and credit risk.** Firms differ in productivity, $A^j$ (type) and $\zeta$ (a smaller $\zeta$ indicates stronger liquidity needs in crises). $\zeta$ drives a wedge between firm type and credit quality. A type-$H$ firm may default by failing to survive, which happens with probability $1-F(x_t^H(\zeta)+\zeta)$. A type-$H$ firm may also strategically default. In contrast, a type-$L$ firm that survives the liquidity crisis may choose to repay its debt if its $\zeta$ is above the strategic default threshold, $\underline{\zeta}_t^L$. This distinction between firm productivity (type) and credit risk is consistent with the evidence that we present in Appendix C.1. To summarize, in both data and the model, firm quality and credit risk are not perfectly aligned. Therefore, although the government may differentiate credit quality by following the private-sector pricing of credit risk, it cannot obtain a perfect indicator of firm productivity from credit markets. Moreover, some credit programs do not even differentiate firms by credit risk, for example, PPP during the Covid-19 crisis (see Appendix B.3 for our analysis).

## 2.3 Efficiency and Equilibrium

**Intervention and efficiency.** The efficiency criterion in our model is whether firms' spending on survival is at the first-best level given by (15). There are two frictions, and both contribute to the financial constraints on spending. The first friction is on the funding supply side, i.e., the limit on private-sector funding, denoted by $\bar{d}$. The second is on the funding demand side, i.e., borrowers' strategic default, which gives rise to the endogenous limit on private-sector debt capacity, $\hat{d}_t^j(\zeta)$.

For firms that repay debts (i.e., with $\zeta \geq \underline{\zeta}_t^j$), government intervention enlarges financing capacity and unequivocally improves efficiency by relaxing the funding constraint. These firms repay their debts after surviving the crisis, so, under the disciplinary effect of debt repayment, they do not over-spend. They spend up to $x_t^{*j}(\zeta) = (\bar{\zeta}_t^j - \zeta)^+$, a level that balances the marginal benefit of improving the survival probability and the marginal cost of resources (see Lemma 1).

For firms that strategically default (i.e., with $\zeta < \underline{\zeta}_t^j$), government intervention also enlarges financing capacity and increases the financing amount, $x_t^j(\zeta)$, but the efficiency implications are more complex. Government intervention enlarges firms' financing capacity through two channels, direct liquidity injection and private funding crowding-in. First, an increase in $\bar{g}$ provides more funding to these firms, as shown in (19). This additional source of funds increases the survival probability and thus lowers the interest rate charged by private-sector creditors as determined by (13). Second, as shown in (18), an increase in $\bar{g}$ raises the survival probability, making the creditors more willing to lend, resulting in a higher $\hat{d}_t^j(\zeta)$ (a crowding-in effect).

**Lemma 4 (Private funding crowding-in)** *For any $\zeta$ and $j \in \{H, L\}$, and given $q_t^j$, government financing crowds in private lending, $\partial \hat{d}_t^j(\zeta)/\partial \bar{g} > 0$, and reduces interest rate, $\partial r_t^j(\zeta)/\partial \bar{g} \leq 0$.*

To analyze the efficiency implication of credit intervention among firms that strategically default, first notice that the goal of these firms is to maximize the survival probability. After surviving the crisis, firm owners renege on debt repayment and extract rent by holding up the creditors in renegotiation. These firms over-spend if the amount they borrow, $\min\{\hat{d}_t^j(\zeta), \bar{d}\} + \bar{g}$, is above the first-best level, $x_t^{*j}(\zeta) = (\bar{\zeta}_t^j - \zeta)^+$, and any further increase in $\bar{g}$ exacerbates such inefficiency.

Among firms that strategically default, over-spending tends to happen to those with high $\zeta$. Formally, if there exists a threshold $\zeta_t^{*j} < \underline{\zeta}_t^j$ such that $\min\{\hat{d}_t^j(\zeta_t^{*j}), \bar{d}\} + \bar{g} = (\bar{\zeta}_t^j - \zeta_t^{*j})^+$ then any firm with $\zeta \in (\zeta_t^{*j}, \underline{\zeta}_t^j)$ over-spends, because the endogenous debt capacity, $\hat{d}_t^j(\zeta)$, is increasing in $\zeta$ while the first-best level of financing, $(\bar{\zeta}_t^j - \zeta)^+$, is decreasing in $\zeta$. For these firms, a higher $\bar{g}$ exacerbates over-spending on survival. For firms that have $\zeta < \zeta_t^{*j}$ and under-spend, an increase in $\bar{g}$ improves efficiency by alleviating their under-spending, whether the firms strategically default or not. The next proposition summarizes the efficiency implications of credit intervention.

**Proposition 2 (Intervention and efficiency)** *Intervention improves efficiency by alleviating under-spending on survival for firms that repay debts (i.e., $\zeta \geq \underline{\zeta}_t^j$) and face a binding funding constraint (i.e., $\bar{d} + \bar{g} < \bar{\zeta}_t^j - \zeta$) and also for firms that strategically default (i.e., $\zeta < \underline{\zeta}_t^j$) and under-spend (i.e., $\hat{d}_t^j(\zeta) + \bar{g} < \bar{\zeta}_t^j - \zeta$). Intervention reduces efficiency by exacerbating over-spending on survival among firms that strategically default (i.e., $\zeta < \underline{\zeta}_t^j$) and over-spend (i.e., $\hat{d}_t^j(\zeta) + \bar{g} > \bar{\zeta}_t^j - \zeta$).*

It is clear that the detrimental effects of credit intervention through over-spending depends on the scale of intervention. When the government increases $\bar{g}$, the spending of firms that strategically default increases through higher $\bar{g}$ and through a higher $\hat{d}_t^j(\zeta)$ due to the crowding-in effect in Lemma 4. This enlarges the over-spending region of $\zeta$ and thus exacerbates the inefficiency from over-spending. On the other hand, when $\bar{g}$ declines, the over-spending region shrinks.

Moreover, a sufficiently large scale of intervention can guarantee a net negative impact. There is a limit to efficiency improvement from relaxing the financial constraint on under-spending firms: When $\bar{g}$ is sufficiently large, all firms that repay debts have reached their first-best level of financing and would not spend more. However, there is no limit on over-borrowing and over-spending: The goal of firms that strategically default is to maximize their survival probability, so they always borrow as much as they can from the government. The following corollary summarizes this result.

**Corollary 1 (Excessive credit intervention)** *The cost of over-spending dominates the benefit from financing the under-spending firms when the scale of credit intervention is sufficiently large.*

So far, we have separately discussed the efficiency implications of credit intervention on firms with $\zeta \geq \underline{\zeta}_t^j$ and on firms with $\zeta < \underline{\zeta}_t^j$. Credit intervention may also affect the strategic default threshold, $\underline{\zeta}_t^j$. While a higher $\bar{g}$ does not affect $\beta$, the value that firms owners can extract through strategic default and renegotiation, it increases firm value in the debt repayment region by relaxing the financial constraint, allowing profitable investment to be funded.[23] This force lowers the default threshold, $\underline{\zeta}_t^j$ and reduces the measure of firms that strategically default. However, as we will show in the next section, this channel of efficiency improvement is not quantitatively significant.

We have analyzed the efficiency implications of intervention within a firm type. Lemma 1, 2, and 3 also reveal the difference in the impact of intervention across firm types. In Proposition 2, credit intervention improves efficiency by relaxing the financial constraint on firms that repay debts. According to Lemma 1, among these firms, type-$H$ firms have a higher first-best level of spending so this positive impact is more prominent among type-$H$ firms. In Proposition 2, credit intervention exacerbates over-spending among firms that strategically default. According to Lemma 2, there are more type-$L$ firms that strategically default than type-$H$ firms. This suggests that the negative impact through over-spending tends to be more reflected among type-$L$ firms.

By incorporating the funding supply- and demand-side frictions, our model allows both under- and over-spending to emerge in equilibrium. While under-spending can happen to both firms that repay debts and those that strategically default, over-spending happens to the latter. Credit intervention improves efficiency among firms that under-spend and exacerbates inefficiency among those that over-spend. The goal of our model is to capture such intricate effects of credit intervention and, after calibration, to allow the dominant channels to emerge in the quantitative analysis.

[23]Recall that once the firm survives the liquidity shock, a strategic default is a comparison between the firm value under debt repayment, which increases in $\bar{g}$, and the value from strategic default $\beta$.

In our model, credit intervention is size-dependent but not dependent on an individual firm's type, $j \in \{H, L\}$, or the severity of its liquidity crisis, $\zeta$. A firm with $k_t$ units of capital can borrow from the government up to $k_t\bar{g}$. There are several motivations behind this setup. First and foremost, the government may not have information on firms' types, in line with the tradition in economics that emphasizes the informational disadvantage of central authorities (Hayek, 1945). Importantly, our setup captures policy design in reality. The funding limit is set proportional to accounting measures of operation scale ($k_t$ in our model) such as the programs during Covid-19 pandemic. MSLP set a borrowing limit to six times the borrower's operating income rather than profits or productivity per unit of resources deployed. Similarly, the limit in PPP was a multiple of the borrower's payroll rather than tied to labor productivity. Furthermore, the corporate bond purchase programs, i.e., PMCCF and SMCCF, imposed limits proportional to an issuer's existing debts (a measure of liability size rather than productivity).[24]

In the next subsection, we discuss how the differential effects of credit intervention on type-$L$ and $H$ firms translate into channels that may affect the cleansing effect of crises and thereby generate intertemporal dependence in intervention scale across crises (i.e., the slippery slope of intervention). In the following, we close this subsection and complete the equilibrium characterization by defining the stationary equilibrium and solving the endogenous capital values.

**Capital value and equilibrium.** Capital value plays an important role. It is the key distinguishing factor that separates type-$L$ and type-$H$ firms. As shown in Lemma 1, capital values drive the first-best level of spending in crises. According to Lemma 2 and 3, capital values affect the strategic default thresholds and firms' endogenous debt capacity in the private funding market.

To simplify notation, we define the expected value per unit of type-$j$ capital to the owner of a type-$j$ firm with a realized $\zeta$ as follows

$$\pi_t^j(\zeta) = \underbrace{\left(F(x_t^j(\zeta)+\zeta)q_t^j - x_t^j(\zeta)\right)\mathbf{1}_{\zeta \geq \underline{\zeta}_t^j}}_{\text{no strategic default}} + \underbrace{F(x_t^j(\zeta)+\zeta)\beta\mathbf{1}_{\zeta < \underline{\zeta}_t^j}}_{\text{strategic default}}, \tag{20}$$

where $F(\cdot)$ is the survival probability as a function of crisis severity, $\zeta$, and funds available $x_t^j(\zeta)$.

[24]Primary market corporate bond purchase facility: The amount of outstanding bonds or loans of an eligible issuer that borrows from the Facility may not exceed 130 percent of the issuer's maximum outstanding bonds and loans on any day between March 22, 2019, and March 22, 2020. Secondary market corporate bond purchase facility: The amount of bonds that the Facility will purchase from the secondary market of any eligible issuer is capped at 10 percent of the issuers maximum bonds outstanding on any day between March 22, 2019, and March 22, 2020.

Note that, in case of no strategic default, we have substituted out the interest rate using the creditors' break-even condition (18). In equilibrium, capital value, $q_t^j$, satisfies the following equation that equates investors' required rate of return over $dt$, i.e., the discount rate, $rdt$, and the sum of expected return on holding capital from price appreciation, cash-flow yield net off investment costs in normal times, capital growth via investment net off depreciation, and return in crises calculated as the difference between the $\zeta$-averaged capital value in a crisis and the pre-crisis capital value:

$$r = \mathbb{E}_t\left[\frac{dq_t^j/dt}{q_{t-}^j}\right] + \frac{A^j - \phi(\iota_{t-}^j)}{q_{t-}^j} + (\iota_{t-}^j - \delta) + \lambda\frac{\int_0^\infty \pi_t^j(\zeta)dH(\zeta) - q_{t-}^j}{q_{t-}^j}, \tag{21}$$

where we use the subscript $t-$ in $q_{t-}^j$ to denote the pre-crisis capital value and accordingly $\iota_{t-}^j$ to denote the investment rate driven by $q_{t-}^j$ (see (7)). The value $\pi_t^j(\zeta)$ is defined in (20), and as reminder, $H(\cdot)$ is the c.d.f. for $\zeta$. The equilibrium can be characterized below.

**Proposition 3 (Stationary equilibrium)** *There exists a stationary equilibrium where firms' investment rate in normal times, $\iota^j$, borrowing in crises, $x^j(\zeta)$, borrowing cost in crises, $r^j(\zeta)$, debt capacity under strategic default, $\hat{d}^j(\zeta)$, and capital value, $q^j$, are time-invariant. Furthermore, the equilibrium is unique with $q^H > q^L$, under the following conditions: (1) $\iota^j - \delta \leq r$; (2) $\frac{\beta\lambda_F}{4[1-\lambda_F(q^j-\beta)]} < 1$, where $\lambda_F$ is the intensity parameter of exponential distribution c.d.f. $F(\cdot)$.*

The first parameter restriction is standard in the asset pricing literature: $\iota^j - \delta$, the growth rate from investment net off depreciation cannot be higher than the discount rate, $r$; otherwise capital value is infinite. The second parameter restriction limits the degree of moral hazard: Given $\lambda_F$ in $F(x_t^j(\zeta) + \zeta) = 1 - e^{\lambda_F(x_t^j(\zeta)+\zeta)}$, the firm owners' bargaining power in strategic default cannot be too large, i.e., $\beta$ cannot be too large relative to $q^j - \beta$ (the creditors' recovery value). And, given the degree of moral hazard, the survival probability cannot be too responsive to liquidity injection, i.e., $\lambda_F$ cannot be too large. We will show that the parameter restrictions (1) and (2) do not bind in our calibration, that is, the interior values deliver close match between model and data moments.

In our baseline model, the scale of credit intervention, $\bar{g}$, is time-invariant. We characterize an equilibrium with constant capital values as it provides a transparent presentation of key economic mechanisms. Later we will consider an economy where the government employs a dynamic strategy of credit intervention, in which case the value of capital varies over time.

## 2.4 The Cleansing Effect of Crises and Slippery Slope of Intervention

In this subsection, we characterize the cleansing effect of crises, which is key to the intertemporal linkage that generates the slippery slope of credit intervention. To characterize the aggregate dynamics, we use (8) and (9) to derive the law of motion for two state variables, capital quality $\omega_t$ defined in (3), and capital quantity $K_t$ defined in (4):

$$\frac{dK_t}{K_{t-}} = \underbrace{\left[-\delta + \left(\omega_{t-}\iota_{t-}^H + (1-\omega_{t-})\iota_{t-}^L\right) + \eta\right]}_{\mu_t^K(\omega_{t-})} dt + \underbrace{\left(\omega_{t-}\kappa_t^H + (1-\omega_{t-})\kappa_t^L - 1\right)}_{\Delta_t^K(\omega_{t-})} dN_t\,, \tag{22}$$

where $\kappa_t^j$, the fraction of type-$j$ capital that survives a crisis, is defined as

$$\kappa_t^j \equiv \int F(x_t^j(\zeta) + \zeta) dH(\zeta), \tag{23}$$

and

$$d\omega_t = \underbrace{\omega_{t-}\left(1-\omega_{t-}\right)\left(\iota_{t-}^H - \iota_{t-}^L - \frac{\eta}{1-\omega_{t-}}\right)}_{\mu_t^\omega(\omega_{t-})} dt + \underbrace{\left(\frac{\omega_{t-}\kappa_t^H}{\omega_{t-}\kappa_t^H + (1-\omega_{t-})\kappa_t^L} - \omega_{t-}\right)}_{\Delta_t^\omega(\omega_{t-})} dN_t. \tag{24}$$

From Proposition 3, the normal-time capital growth rate, $\iota_{t-}^j$, and capital surviving rate in crises, $\kappa_t^j$, are time-invariant in the stationary equilibrium. Given (7), we have $\iota^H > \iota^L$ under $q^H > q^L$, which then implies that $\omega_t$ has a tendency to drift upward. As previously discussed, we preserve the stationarity of $\omega_t$ via the exogenous entry rate of type-$L$ firms, $\eta$, introduced in (9).

The cleansing effect refers to an increase in $\omega_t$ in crises, i.e., $\Delta_t^\omega(\omega_{t-}) > 0$, which is equivalent to type-$H$ firms having a higher survival rate than type-$H$ firms, i.e., $\kappa_t^H > \kappa_t^L$. Therefore, the cleansing effect of crises is simply about which type of firms in aggregate spend more on survival. Consider the first-best scenario where firms are not financially constrained and spend on survival at the level given by (15). Here the cleansing effect emerges because type-$H$ firms have a higher first-best level of spending than type-$L$ firms, i.e., $x^{H*}(\zeta) > x^{L*}(\zeta)$ (see Lemma 1). Type-$H$ firms want to borrow more and spend more on survival because because their capital is more valuable.

**Proposition 4 (The cleansing effect in the first-best economy)** *In the first-best economy where firms spend at the level given by (15), crises have a cleansing effect, i.e., $\Delta_t^\omega(\omega_{t-}) > 0$.*

As previously discussed, once we introduce the funding supply- and demand-side frictions, firms may under- or over-spend on survival. Next, we first discuss how introducing only the funding-supply friction to the first-best economy affects the cleansing effect of crises. The funding supply-side friction, that is the private-sector creditors cannot lend beyond $\bar{d}$, causes firms of both types to under-spend and thereby dampens the cleansing effect in Proposition 4. Intuitively, when this funding-supply constraint binds, i.e., $\bar{d} < x^{j\star}(\zeta)$, both types of firms borrow and spend at the same level, $\bar{d}$. Intervention, by expanding funding supply and allowing more firms to spend at the first-best level, reverses this dampening effect. The next lemma summarizes this result.

**Corollary 2 (Funding-supply friction and the cleansing effect)** *Introducing a sufficiently tight funding constraint ($\bar{d} \to 0$) to the first-best economy weakens the cleansing effect of crises. Credit intervention, by expanding funding supply, reverses this force and strengthens the cleansing effect.*

Next, we discuss the impact of introducing funding demand-side friction on the cleansing effect of crises. The demand-side friction, i.e., firms' inability to commit against strategic default, gives rise to a new channel of cleansing effect. As shown in Lemma 3, firms that strategically default borrow as much as they can to maximize their survival probability, and as a result, the private-sector creditors impose a debt limit. Type-$H$ firms face a higher endogenous debt limit, i.e., $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$ and thus can borrow more than type-$L$ firms. This is a new channel of cleansing effect.[25] While the cleansing effect in Proposition 4 arises from type-$H$ firms wanting to spend more on survival than type-$L$ firms, this new channel is about type-$H$ firms being able to obtain more funding. Credit intervention dampens this channel of cleansing effect by offering funding to both types of firms. Intuitively, under a concave $F(\cdot)$, the marginal impact of $\bar{g}$ on survival probability is greater for type-$L$ firms because type-$H$ firms already have a higher level of private-sector funding.

**Corollary 3 (Funding-demand friction and the cleansing effect)** *Given any measure of $\zeta$ where both types of firms strategically default and the private-sector debt limit binds before the private-sector funding supply constraint (i.e., $\hat{d}_t^j < \bar{d}$), a cleansing effect emerges in crises, i.e., the share of surviving firms of type $H$ is greater than that of type $L$. Credit intervention weakens this channel.*

In sum, two channels of cleansing effects emerge in crises: (1) Among firms that repay debts, type-$H$ firms want to borrow more to preserve capital; (2) among firms that strategically default,

[25] Appendix C.2 provides evidence that type-$H$ firms have larger financing capacity than type-$L$.

type-$H$ firms can borrow more. Credit intervention strengthens the first channel of cleansing effect while can weaken the second channel. However, there is a limit to the impact of credit intervention on the first channel: If $\bar{g}$ is sufficiently large to allow the first-best spending on survival for both types, then any further increase in $\bar{g}$ no longer changes the survival probability for firms that repay debts. In contrast, there is no limit to the impact of credit intervention on the second channel: No matter how large $\bar{g}$ is, any further increase in $\bar{g}$ always improve the survival probability for firms that strategically default as they maximize their borrowing. Therefore, for a sufficiently large $\bar{g}$, the weakening of cleansing effect by credit support for firms that strategically default dominates the strengthening of cleansing effect by credit support for firms that repay debts.

So far, we have discussed that a type-$H$ firm has a higher survival probability than a type-$L$ firm if either both firms repay their debts, in which case type-$H$ firms want to borrow more under a higher spending target, or if both firms strategically default, in which case type-$H$ firms can borrow more under a higher debt capacity. There is also the case where a type-$H$ firm repays its debt while the type-$L$ firm with the same $\zeta$ strategically defaults.[26] In the following, to sharpen the analytical results, we introduce a parameter restriction so that we can focus on characterizing the quantitatively relevant region of parameter values. The following restriction is not a binding constraint in our parameter calibration; in other words, we do not impose the restriction in calibration, but we find that the restriction is satisfied in our baseline calibration and holds in a much broader set of parameter values. We will discuss calibration in details in the next section.

**Condition 1** *$q^H$ is sufficiently large and $q^L$ is sufficiently low so that the equilibrium has the following properties: (1) $\underline{\zeta}^H = 0$, (2) $\bar{\zeta}^L < \bar{d} + \bar{g}$, and (3) $\bar{\zeta}^H > \hat{d}^L(\underline{\zeta}^L) + \underline{\zeta}^L + \bar{g}$.*

Condition 1 focuses on capital values. As discussed in Subsection 2.3, $q^j$ drives $\bar{\zeta}^j$ and, given the severity of a firm's liquidity crisis, $\zeta$, it determines the first-best level of spending through $\bar{\zeta}^j$. Moreover, $q^j$ determines the strategic default threshold $\underline{\zeta}^j$ and the endogenous debt limit, $\hat{d}^j(\zeta)$.

This condition is sufficiently general, as one can view the binary types in our model as a discretized version of a continuum of firm types, and type-$H$ firms represent the right tail of the most productive while the firms with low productivity are represented by type $L$ in the model.

Under Condition 1, type-$H$ firms do not strategically default ($\underline{\zeta}^H = 0$) because the firm owners find it rather costly to lose control right and capital value. Firms may still face a lack of funding

[26]Note that from Lemma 2, type-$H$ firms have a lower default threshold for $\zeta$, so given the same $\zeta$, we can rule out the case where the type-$H$ firm strategically defaults while the type-$L$ firm repays its debt.

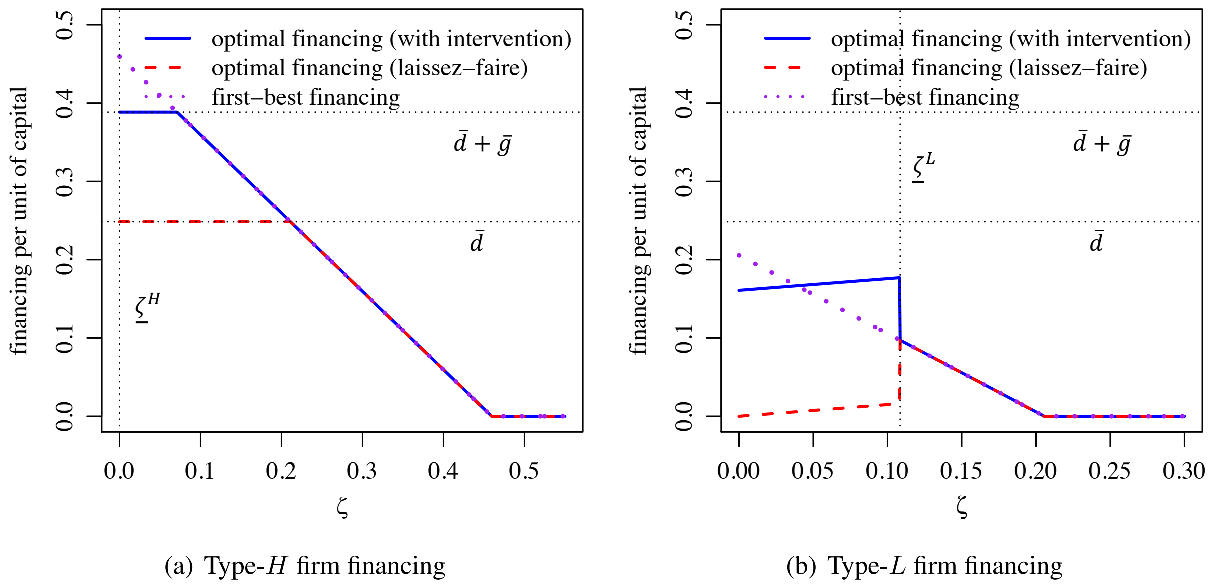



**Figure 1: Optimal firm financing.** This figure illustrates the optimal firm financing in different scenarios, under the calibrated model parameters that we discuss in Section 3. The solid line shows the optimal financing under credit intervention. The dashed line shows the optimal financing without credit intervention ($\bar{g} = 0$). The dotted line shows the first-best financing amount without funding supply- and demand-side frictions.

supply, so, for type $H$, the relevant form of inefficiency is under-spending. Panel A of Figure 1 illustrates the optimal amount of financing for a type-$H$ firm with different values of $\zeta$. This graph is based on the calibrated parameter values that will be discussed in the next section. When a firm has a sufficiently large $\zeta$, the first-best level of financing is zero (see (15)). As $\zeta$ declines, the liquidity crisis becomes more severe, and the first-best level of financing increases (dotted line). Financing is constrained by funding supply, which is equal to $\bar{d}$ without government intervention (dashed line) and is equal to $\bar{d} + \bar{g}$ with government intervention (solid line).

The second property, $\bar{\zeta}^L < \bar{d} + \bar{g}$, implies that type-$L$ firms do not under-spend due to the lack of funding supply even when the liquidity shock is most severe ($\zeta = 0$ and the first-best spending is $\bar{\zeta}^L$). This is due to the fact that a low value of type-$L$ capital leads to a low first-best spending level. As shown in Panel B of Figure 1, the first-best level (dotted line) is below $\bar{d} + \bar{g}$. Note that type-$L$ firms still face the endogenous debt limit due to strategic default. The dashed line in Panel B of Figure 1 shows that when $\zeta$ is small and the liquidity needs are strong, the endogenous debt limit binds. As $\zeta$ increases from zero, the debt limit, $\hat{d}^L(\zeta)$, increases (see Lemma 3), so the firm is able to raise more funds. However, as long as $\zeta$ is below the strategic default threshold, $\underline{\zeta}^L$, the financing level is below the first-best. The solid line shows the situation with credit intervention. The firm is able to raise more funds from the government and from the private-sector creditors due to the crowding-in effect in Lemma 4. However, over-spending emerges among firms with relatively high $\zeta$ (still below $\underline{\zeta}^L$), even though those with $\zeta$ close to zero still under-spend.

As previously discussed, crises generate a cleansing effect among firms that repay debts and among firms that strategically default. However, given the same $\zeta$, when type-$H$ firms repay debts and type-$L$ firms strategically default (i.e., $\zeta \in (\underline{\zeta}^H, \underline{\zeta}^L)$), it is ambiguous which type spends more. The last property delivered by Condition 1 resolves such ambiguity. For any $\zeta < \underline{\zeta}^L$ (type-$L$'s strategic default region), the last property implies $\bar{\zeta}^H - \zeta > \hat{d}^L(\underline{\zeta}^L) + \bar{g}$ so that type-$H$ firms spend more on survival than type-$L$ firms, which leads to the cleansing effect of crises in this region of $\zeta$.

**Proposition 5 (Capital destruction and the cleansing effect in crises)** *Crises feature capital destruction, $\Delta_t^K < 0$, and under Condition 1, have a cleansing effect, $\Delta_t^\omega > 0$.*

Our focus on crises with cleansing effects is empirically motivated. For example, during the Covid-19 crisis, firms with lower productivity were more likely to cease operations permanently (Muzi et al., 2023). And, during the Covid-19 crisis, economic activity was reallocated toward firms with higher pre-crisis labor productivity, and such reallocation is stronger compared with pre-crisis times (Bruhn et al., 2023). These studies also document that credit intervention dampens the cleansing effect. Moreover, Dörr et al. (2022) find that credit intervention during the Covid-19 pandemic disproportionately benefited firms that are already financially vulnerable pre-crisis.[27] Consistent with the evidence, the proposition shows that credit intervention dampens the cleansing effect in our model. Moreover, credit intervention has a positive effect on firm survival and the total quantity of capital preserved, which is also consistent with the evidence (Bartik et al., 2020; Bartlett and Morse, 2020; Hubbard and Strain, 2020; Denes et al., 2021; Kawaguchi et al., 2021). Therefore, the government faces a trade-off between capital quantity, $K_t$, and quality $\omega_t$.

**Proposition 6 (The quantity-quality trade-off)** *Credit intervention alleviates capital destruction*

$$\frac{\partial \Delta_t^K}{\partial \bar{g}} > 0,$$

[27]Muzi et al. (2023) analyze data from firms in 34 countries. Bruhn et al. (2023) utilizes the World Banks Enterprise Surveys Covid-19 Follow-up Surveys, encompassing around 8,000 firms in 23 emerging and developing countries across Europe and Asia. Dörr et al. (2022) examine 1.5 million German companies.

*but dampens the cleansing effect*

$$\frac{\partial \Delta_t^\omega}{\partial \bar{g}} < 0.$$

**The slippery slope of intervention.** The cleansing effect of crises, $\Delta_t^\omega > 0$, and the impact of government intervention, $\partial \Delta_t^\omega / \partial \bar{g} < 0$ are important for analyzing the intertemporal linkages. Firm quality, $\omega_t$, is a key state variable. By dampening the cleansing effect of the current crisis, credit intervention begets an intervention of greater scale in the next crisis. The following proposition characterizes a critical connection between pre-crisis firm quality ($\omega_{t-}$) and intervention scale.

**Proposition 7 (Pre-crisis firm quality and intervention scale)** *To contain the output drop at any given level, the required scale of intervention is larger if the pre-crisis firm quality is lower.*

Output drop in crises is caused by the decline of capital quantity. Since type-$L$ firms have a lower capital productivity, they need more units of capital to produce the same level of output than type-$H$ firms. Furthermore, type-$L$ firms have less private-sector financing capacity so they need to rely more on the government. Therefore, when an economy enters into a crisis with more type-$L$ firms and less type-$H$ firms (i.e., a lower $\omega_{t-}$), it requires the government to provide a larger scale of intervention if the goal is to contain the drop in output at a certain level.[28]

This connection between the pre-crisis firm quality and intervention scale is key to establish the slippery slope of intervention. In Proposition 6, we show that credit intervention dampens the cleansing effect and reduces the post-crisis average firm quality, $\omega_t$. When the economy enters into the next crisis with a lower firm quality, the scale of intervention has to increase, if the policy maker aims to contain the output drop at any given level. In the next section, we calibrate our model and show that the slippery slope of intervention is quantitatively important.

**Discussion: Low productivity firms vs. zombie firms.** Zombie firms are permanently impaired firms whose operation relies on external financial resources (Caballero et al., 2008; Acharya et al., 2019, 2021). Type-$L$ firms are not zombies. In a crisis, both type-$H$ and $L$ firms may over-spend and can thus be viewed as engaging in a negative-NPV transaction, but after they survive the crisis, these firms will spend at the efficient level in normal times, guided by their Tobin's q (capital value), and use their capital to produce. Firms' problem in crises is of a temporary nature. We

[28] This mechanism is still valid when the policy goal is to contain capital destruction (see Appendix D.3).

model a liquidity crisis, not a solvency crisis. Relative to the literature on zombie firms, our paper offers a different and complementary perspective on the distortions from credit intervention. In Appendix B.4, we discuss how to incorporate zombie firms in our model and how the presence of such firms amplifies the distortionary effects of policy intervention. Finally, we emphasize that our mechanism does not rely on zombies crowding out normal firms in product or factor markets, which is a key ingredient in models on crisis cleansing effect, zombie firms, and the efficiency implications of policy intervention (e.g., Caballero and Hammour, 1994; Acharya et al., 2021).

# 3 Quantitative Analysis

## 3.1 Model Calibration

We define the average value of $\omega_t$ as $\bar{\omega}$, which is long-run average implied by the stationary distribution of $\omega_t$, determined by both the drift and Poisson components of law of motion. For our simulation analysis, we will initiate $\omega_t$ at this average value.

To calibrate the model, first, we parameterize the survival probability function as $F(x) = 1 - \exp(-\lambda_F x)$ and, the for the cumulative distribution function of $\zeta$ that indexes the baseline level of survival probability, we use $H(\zeta) = 1 - \exp(-\zeta/l_\zeta)$ (with an average $\zeta$ equal to $l_\zeta$). Including $\lambda_F$ and $l_\zeta$, we have a total of twelve parameters. The other ten parameters include the investment cost parameter $\theta$, crisis frequency $\lambda$, debt restructuring rent $\beta$, capital depreciation rate $\delta$, discount rate $r$, capital productivities $A^H$ and $A^L$, entry rate of new firms $\eta$, the private-sector credit-supply limit in crises $\bar{d}$, and a baseline value for government credit support $\bar{g}$. The parameter values and moment conditions are summarized in Table 1 and Table 2, respectively.

The investment cost function controls the average investment rate of the economy, and we set the parameter $\theta$ to generate an average annual investment-to-capital ratio of 10% following the literature (e.g., Gertler et al. (2020) target a 2.5% quarterly investment-to-capital ratio).

We set the parameter governing the survival probability function, $\lambda_F$ to generate an average output drop in crises that matches the average GDP decline in crises. According to Reinhart and Rogoff (2009), the peak-to-trough decline in GDP across a large sample of crises is 9.3%.

The parameter $l_\zeta$ in the distribution of $\zeta$ affects the sensitivity of firm survival to credit intervention. When $l_\zeta$ is small, there is a large density of type-$L$ firms that rely on government funding support, so the impact of intervention on firm survival is stronger. Bartlett III and Morse (2020)

**Table 1: Parameter values**. This table shows the parameter values in our quantitative analysis and the corresponding calibration targets. Five model parameters are directly set by the observed data, including $\lambda$, $\delta$, $r$, $\eta$, and $\bar{g}$. The rest of parameters, $\theta$, $\lambda_F$, $l_\zeta$, $\beta$, $A^H$, $A^L$, and $\bar{d}$ are solved to match the moment targets.

| Parameter | Description | Value | Data Source or Targeted Moment |
|---|---|---|---|
| $\theta$ | Investment cost | 9.2 | Average investment/capital ratio |
| $\lambda_F$ | Survival rate | 5.8 | Average GDP drop in crises |
| $l_\zeta$ | Average $\zeta$ | 0.16 | Impact of credit intervention on firm survival |
| $\lambda$ | Crisis frequency | 0.06 | Crisis frequency in the data |
| $\beta$ | Debt restructuring rent | 0.43 | Average creditor recovery rate |
| $\delta$ | Capital depreciation rate | 0.2 | Capital depreciation and firm exit rate |
| $r$ | Real discount rate | 0.06 | Average real bond return plus equity premium |
| $A^H$ | Productivity of type-$H$ firms | 0.57 | Average output-to-capital ratio |
| $A^L$ | Productivity of type-$L$ firms | 0.15 | TFP inter-quartile ratio |
| $\eta$ | Entry rate of new firms | 0.062 | Firm entry rate |
| $\bar{d}$ | Private-sector credit availability | 0.25 | Private-sector debt/GDP ratio |
| $\bar{g}$ | Government credit support | 0.14 | Covid-19 credit support in the U.S. |

**Table 2: Moment matching**. We report the moment matching results for calibrating $\theta$, $\lambda_F$, $l_\zeta$, $\beta$, $A^H$, $A^L$, and $\bar{d}$.

| Moment Description | Model | Data |
|---|---|---|
| Average investment-to-capital ratio | 0.1 | 0.1 |
| Average GDP drop in crises | -9.3% | -9.3% |
| Average impact of credit intervention on firm survival likelihood | 10% | 10% |
| Average creditor recovery rate | 49% | 49% |
| Average output-to-capital ratio | 45% | 45% |
| TFP ratio between 90% and 10% percentiles | 3.7 | 3.7 |
| Average private-sector debt/GDP ratio | 36% | 36% |

used a survey of 1,000 small businesses to study how PPP loans impacted their survival likelihoods. They find that PPP application success increased a firm's medium-run survival probability by 20.5%, but only for microbusinesses (those with 1-5 employees). Kawaguchi et al. (2021) surveyed small businesses during the Covid-19 pandemic and found that lump-sum subsidies in Japan increased firms' self-reported prospects of survival by 19%. However, there is also evidence that the impact is much smaller. For example, Hubbard and Strain (2020) used an intent-to-treat model with data on private firms from Dun & Bradstreet. They find that PPP eligibility reduced business closure odds by 0.22% with no significant effect for firms closer to the 500-employee cutoff. Given the mixed evidence, we target a 10% survival likelihood improvement.

Next, for crisis frequency $\lambda$, we map to the empirical counterpart in Taylor (2015), i.e., the frequency of crises is about 6% in a panel of 17 countries from 1800 to 2012.

The parameter $\beta$, which represents the borrowers' value after debt renegotiation and restructuring, governs the firm owners' incentive to strategically default. We set the parameter to generate a bankruptcy recovery rate for creditors that match the empirical counterpart. Below we explain how we calculate the creditors' recovery rate in our model. For type-$j$ firm, $j \in \{H, L\}$, there are two cases of bankruptcy: (1) Liquidity-induced bankruptcy, which happens with probability $1 - F(\zeta + x^j(\zeta))$, and the recovery rate is zero; (2) Strategic default, which happens when a firm survives the crisis and has $\zeta < \underline{\zeta}^j$, generates a recovery rate for creditor equal to $(q^j - \beta)/x^j(\zeta)$. Therefore, the creditors' recovery rate of type $j$ firm in bankruptcy (averaging over $\zeta$) is

$$\mathcal{R}^j = \int \left( (1 - F(\zeta + x^j(\zeta)) \cdot 0 + F(\zeta + x^j(\zeta)) 1_{\zeta < \underline{\zeta}^j} \frac{q^j - \beta}{x^j(\zeta)} \right) dH(\zeta)$$

with a conditional probability mass

$$\mathcal{M}^j = \int \left( (1 - F(\zeta + x^j(\zeta)) + F(\zeta + x^j(\zeta)) 1_{\zeta < \underline{\zeta}^j} \right) dH(\zeta)$$

Then total (cross-type) average creditor recovery rate is

$$\bar{R} = \frac{\bar{w}\mathcal{R}^H + (1 - \bar{w})\mathcal{R}^L}{\bar{w}\mathcal{M}^H + (1 - \bar{w})\mathcal{M}^L}$$

The moment target for $\bar{R}$ is the empirical average creditor recovery in Chapter 11 bankruptcies. According to Antill (2022), average recovery rate from Moody's Ultimate Recovery database is 49%, and according to Dou et al. (2021), the average recovery rates from across different classes of creditors is 46%.[29] We use 49% recovery rate as our targeted value.

The parameter, $\delta$ in the model can be interpreted as the depreciation rate of capital plus exits of businesses in normal times. According to Business Dynamics Statistics, the average firm establishment exit annual shutdown rate is about 10% from 1982 to 2020. With a quarterly depreciation of 2.5% (Gertler and Karadi, 2011b), we set $\delta = 0.025 * 4 + 0.1 = 0.2$.[30]

The discount rate $r$ reflects how the firm owners discount cash flows. Since productive capital is a long-term asset, we consider the long-term real rate plus an unlevered equity premium. In the

[29] See Row 1 of Table 1 in Antill (2022). Table 1 Panel B in Dou et al. (2021) reports junior and senior creditor recovery rates and their fractions of total debt. The weighted average is (0.559)*0.204 + (1-0.559)*0.788 = 0.46.

[30] We note that a larger depreciation parameter $\delta$ in the literature is not uncommon, especially when it incorporates the business exit rate. For example, Gertler et al. (2020) set $\delta = 0.33$ to match the investment to capital ratio.

U.S., the historical average real return of government bond is $2.85\%$ and the historical average real return of equity is $8.46\%$.[31] The proper discount rate on capital is the unlevered cost of equity, which is between the two discount rates, depending on average leverage. With a historical average debt/equity ratio of 0.5, the unlevered real return of equity is $6.6\%$.[32] Therefore, we set $r = 0.066$.

We choose the productivity parameters $A^H$ and $A^L$ jointly to generate a ratio $A^H/A^L$ that matches the ratio between 90% and 10% percentiles of TFP distribution[33] and an average output-to-capital ratio close to 1/2.24 in Elenev et al. (2021).[34] And, we calibrate the firm entry rate to the data counterpart in Clementi and Palazzo (2016), setting $\eta = 0.062$.[35] For $\bar{d}$ that determines private-sector credit availability, we target the ratio of total (non-financial) private-sector debt to GDP in the data. We match the actual private-sector lending to GDP ratio in the model to the data counterpart. This ratio is on average 0.36 in the historical sample of 1952–2023.[36]

Finally, we set the baseline case $\bar{g}$ according to the size of intervention (not total takeup, which is smaller) during Covid-19, including programs initiated by both the central bank and government in the U.S. The Fed set up around 2.3 trillion credit support for firms (c.11% of 2020 GDP) decomposed into 0.6 trillion in MSLP, 0.75 trillion in PMCCF and SMCCF, and 0.95 trillion in PPP Liquidity Facility. The total fiscal response that supports businesses is 4 trillion.[37] Taken together, the amount is about 30% of the GDP in 2020. Thus, we set the ratio of credit support-to-capital stock $\bar{g} = 0.45 * 30\% \approx 0.14$ where, as previously discussed, $0.45$ is the output-to-capital ratio.

## 3.2 The Trade-Off between Quantity and Quality

In Proposition 6, we show that an increase in the scale of credit intervention preserves more units of capital in crises, i.e., $\partial\Delta_t^K/\partial\bar{g} > 0$, but dampens the cleansing effect of crises, i.e., $\partial\Delta_t^\omega/\partial\bar{g} < 0$.

[31] See Tables VII and X of Jordà et al. (2019). Note that the world-average real return of long-term government bond is 2.61%, close to the U.S. average. The world-average real equity return is 7.12%, also similar to the U.S. average.

[32] We download the flow-of-funds data on non-financial corporate business debt (ticker "BCNSDODNS") and equity (ticker "NCBEILQ027S"), and the average debt/equity ratio from 1951-2022 is about 0.5. We ignore tax and calculate the unlevered required return on equity as $(0.0846 + 0.5 \times 0.0285)/(1 + 0.5) = 6.6\%$.

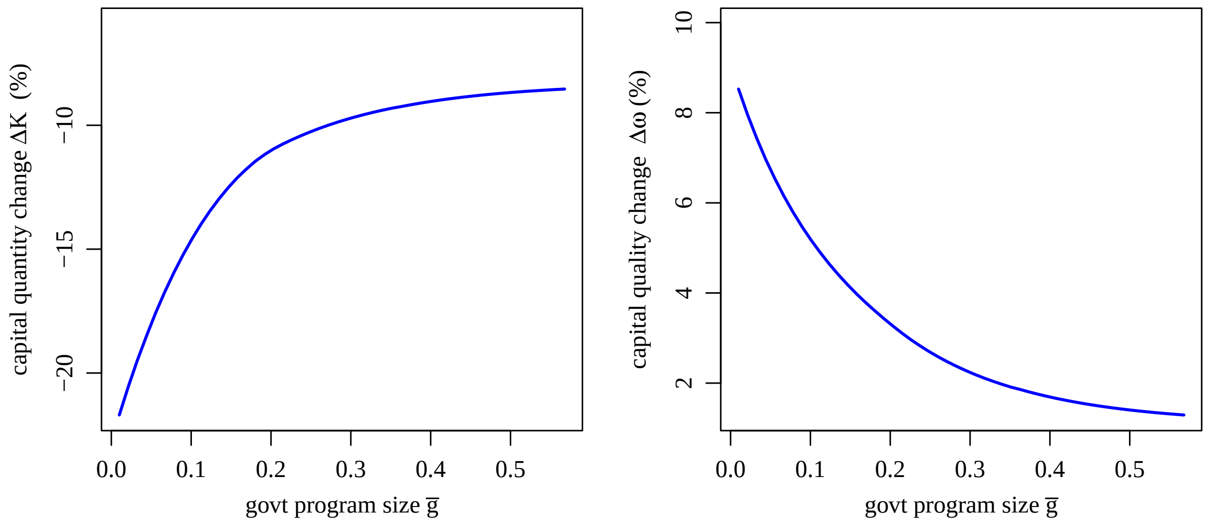
[33] See Appendix C.1 for details of how we measure the TFP distribution and Appendix D.2 for our solution of an extended model where a firm's productivity switches between $A^H$ and $A^L$ at idiosyncratic Poisson times.

[34] An alternative target is the Kaldor facts in growth models, i.e., an average capital to output ratio of 2.5.

[35] In Appendix D.1, we conduct sensitivity analysis for this parameter.

[36] See Table "Debt of Nonfinancial Sectors, 1952–2023" under "Z.1-Financial Accounts" issued by the Fed.

[37] For the decomposition of the programs offered by the Fed, see this Brookings article, "What did the Fed do in response to the Covid-19 crisis?" , and for the decomposition of fiscal reponse, see this IMF report, "Fiscal Monitor Database of Country Fiscal Measures in Response to the Covid-19 Pandemic".



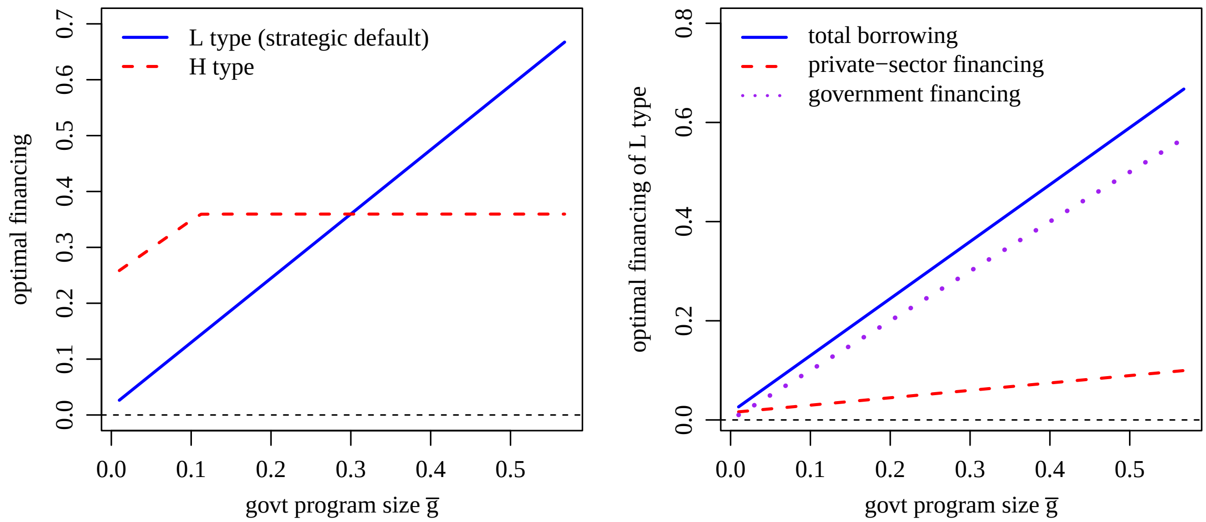

(a) Credit intervention and capital quantity

(b) Credit intervention and capital quality

**Figure 2: Credit intervention impact: Capital quantity vs. quality.** This figure illustrates how $\bar{g}$ affects $\Delta K_t$ and $\Delta\omega_t$ in a crisis. The calculation requires the pre-crisis $\omega_{t-}$ which we set to the average value $\bar{\omega}$ defined in Section 3.1.

Capital quantity, $K_t$, and quality, $\omega_t$ are the two state variables that drive aggregate dynamics. Figure 2 illustrates the quantity-quality trade-off. We fix the pre-crisis $\omega_t$ at the average value $\bar{\omega}$ defined in Section 3.1. In Panel A, we plot the percentage change of $K_t$ against $\bar{g}$. The figure show that as intervention changes from $\bar{g} = 0$ to $\bar{g} = 0.14$ (i.e., 30% of GDP as discussed in Section 3.1), the percentage destruction of capital shrinks from $-22\%$ to $-13\%$. In Panel B, as intervention changes from $\bar{g} = 0$ to $\bar{g} = 0.14$, the cleansing effect $\Delta\omega_t$ falls from about 9% to 4%.

An increase in the scale of intervention, $\bar{g}$, improves efficiency by reducing the gap between financially constrained firms' spending and the first-best level. A higher $\bar{g}$ can also lead to over-spending among firms that strategically default. While both forces preserve capital quantity, their impact on capital quality differs. Lemma 2 shows that there are more type-$L$ firms that choose to strategically default than type-$H$ firms. The calibrated parameter values satisfy Condition 1 in Section 2: Type-$H$ firms do not strategically default, while strategic default and over-spending can happen among type-$L$ firms. Therefore, when $\bar{g}$ increases, its positive impact on type-$H$ firms' survival is limited, as type-$H$ firms do not spend beyond the first-best level. In contrast, among type-$L$ firms that strategically default, spending on survival always increases in $\bar{g}$. Thus, as the size of intervention increases, type-$L$ firms benefit more. This dampens the cleansing effect of crises.




(a) Firm financing: $x_t^L(\zeta)$ and $x_t^H(\zeta)$

(b) Decomposing type-$L$ firm financing

**Figure 3: Credit intervention and firm financing.** This figure illustrates how intervention affects the optimal amount of financing by H and L type firms, accounting for the endogenous responses of $q^H$ and $q^L$. In both panels, we choose $\zeta = 0.1$, which is a case of strategic default for L-type firms. In panel (b), we decompose the actual borrowing of L-type firms into private-sector financing (which reaches the endogenous limit $\hat{d}^L$), and government financing (which reaches the limit $\bar{g}$). As $\bar{g}$ changes, all equilibrium variables change accordingly, including capital values.

Panel A of Figure 3 illustrates this mechanism. Fixing a value of $\zeta$, we plot the optimal financing and spending of a type-$H$ firm (dashed line) against $\bar{g}$, which flattens out at the first-best level, and that of a type-$L$ firm that strategically defaults, which is always increasing in $\bar{g}$. In Panel B, we decompose the type-$L$ firm's spending into funding from the government (dashed line) and the private sector (dotted line). A higher $\bar{g}$ allows the type-$L$ firm to spend more through both the direct liquidity provision and crowding in the private-sector funding (see Lemma 4 in Section 2).

So far, our discussion of the welfare and efficiency implications of credit intervention, and has focused on the impact in crises, i.e., the quantity-quality trade-off, $\partial \Delta_t^K / \partial \bar{g} > 0$ and $\partial \Delta_t^\omega / \partial \bar{g} < 0$. The impact of credit intervention also spills over to the normal times. By improving survival probability for both types, credit intervention increases capital values, $q^L$ and $q^H$, and thereby raises normal-time investment rates, $\iota^H$ and $\iota^L$ (see (7)), lifting upward the growth trajectory of capital quantity, $K_t$. Moreover, as previously discussed, the positive impact on survival probability is greater for type-$L$ firms (which is why credit intervention dampens the cleansing effect of crises), so a higher $\bar{g}$ increases $q^L$ more than $q^H$, introducing a downward bias in the drift of $\omega_t$.

Next, we introduce the welfare function as a criterion for the overall impact of credit intervention. At time $t$, the social welfare is defined as the present value of life-time consumption flows and is a function of the two state variables, $K_t$ and $\omega_t$. Since the economy is scalable with respect to capital, we conjecture the welfare at time $t$ as $W(\omega_t)K_t$. It can be written as follows:

$$\mathbb{E}_t\left[\int_t^{\infty} e^{-r(s-t)}(\omega_s A^H + (1-\omega_s)A^L)K_s ds - \left(\omega_s \iota_s^H + (1-\omega_s)\iota_s^L\right) K_s ds - I_s K_{s-} dN_s\right], \quad (25)$$

where, in the integral, we record the consumption flow as the aggregate output net off goods invested in normal times and crises times, and the spending in a crisis at time $s$, $I_s$, is given by

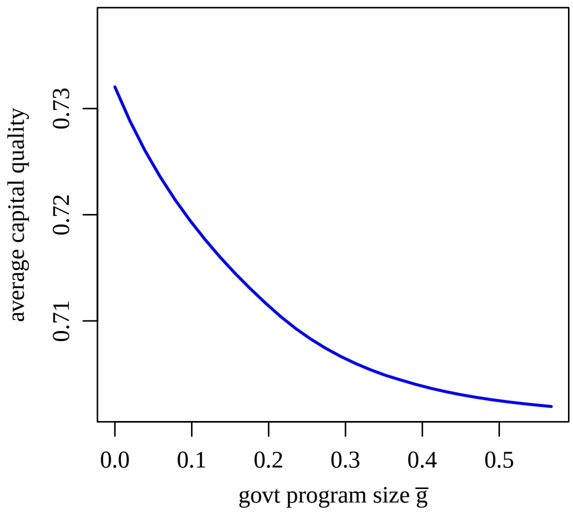
$$I_s \equiv \omega_{s-}\int_\zeta x_s^H(\zeta)dH(\zeta) + (1-\omega_{s-})\int_\zeta x_s^L(\zeta)dH(\zeta). \quad (26)$$

In the stationary equilibrium, the $K_t$-scaled welfare function $W(\omega)$ satisfies the following ordinary differential equation:

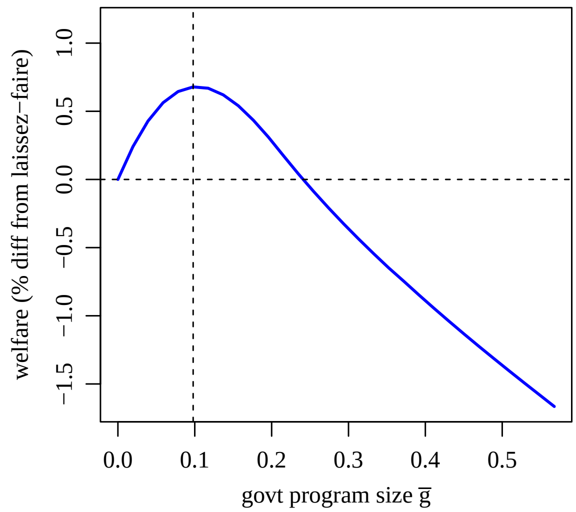
$$\begin{aligned} rW(\omega) &= \omega A^H + (1-\omega)A^L - \left(\omega\iota^H + (1-\omega)\iota^L\right) + W(\omega)\mu_K(\omega) + W'(\omega)\mu_\omega(\omega) \\ &\quad - \lambda I(\omega) + \lambda\left[W(\omega+\Delta^\omega(\omega))\left(1+\Delta^K(\omega)\right) - W(\omega)\right] \end{aligned} \quad (27)$$

We numerically solve for the welfare function $W(\omega)$ as determined by equation (27) and illustrate the impact of $\bar{g}$ on welfare in Figure 4. To highlight the dependence of welfare on $\bar{g}$, we write the welfare as $W(\omega;\bar{g})$. Figure 4(a) shows the impact of $\bar{g}$ on the average firm quality $\bar{\omega}(\bar{g})$, where the average is calculated based on simulated path of the economy. We find that as the government expands its scale of intervention, the average firm quality declines.

Figure 4(b) plots percentage improvement in welfare from the laissez-faire economy to the intervened economy, $W(\bar{\omega}(\bar{g});\bar{g})/W(\bar{\omega}(0);0) - 1$, which increases in $\bar{g}$ for small $\bar{g}$, but decreases once $\bar{g}$ passes a threshold. The welfare curve is upward-sloping when the scale of intervention is low. The marginal improvement of welfare due to type-$H$ firms' efficient spending dominates the loss from wasteful spending by type-$L$ firms that strategically default. Therefore, a timid intervention almost guarantees a positive contribution to welfare at the margin. In Corollary 1 in Section 2, we show that excessive credit intervention can destroy welfare, which is on the downward-sloping component of the welfare curve. The over-spending by type-$L$ firms comes at the expense of aggregate consumption, so even though the total capital stock, $K_t$, grows faster, the social welfare,




(a) Average capital quality




(b) Welfare improvement

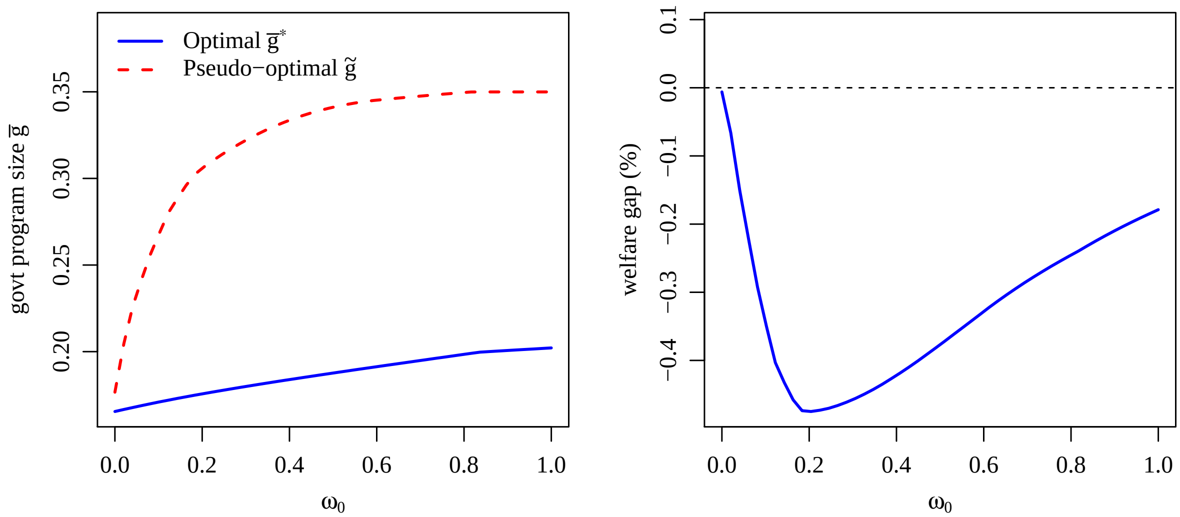
**Figure 4: Credit intervention and welfare.** In panel (a), we plot the average firm quality $\bar{\omega}$ as a function of government intervention $\bar{g}$. For each $\bar{g}$, we solve the model again and calculate the average of simulated $\omega_t$ as $\bar{\omega}(\bar{g})$. In panel (b), we show the welfare difference $W(\bar{\omega}(\bar{g}); \bar{g})/W(\bar{\omega}(0); 0) - 1$ as a function of government intervention $\bar{g}$.

which is the present value of households' life-time consumption, declines. Intuitively, the optimal intervention balances the benefit from relaxing the financial constraint on under-spending firms and the cost of exacerbating over-spending among type-$L$ firms that strategically default. The vertical dotted line marks the optimal intervention size, which is equivalent to 22% of GDP. For comparison, in Section 3.1, we document that the total scale of credit support from the government and central bank in the U.S. during Covid-19 crisis is about 30% of GDP.

In the stationary equilibrium, the scale of intervention, $\bar{g}$, is constant, chosen at $t = 0$. In Panel A of Figure 5, we allow the government to optimize $\bar{g}$ at $t = 0$, i.e.,

$$g^*(\omega_0) = \arg\max_{\bar{g}} W(\omega_0; \bar{g})$$

and plot the optimal $\bar{g}$ against $\omega_0$ (solid line). The curve is upward-sloping. Intuitively, at the left end where the economy is dominated with type-$L$ firms, the government optimally restricts funding support because a large scale intervention is likely to result in type-$L$ firms' over-spending. In contrast, near the right end where type-$H$ firms dominate, the optimal scale of intervention is high. When the economy starts with a larger fraction of firms being type-$H$ (i.e., $\omega_0$ is higher), the




(a) Optimal government intervention

(b) Difference in optimal welfare

**Figure 5: Optimal intervention and welfare.** In panel (a), we plot the optimal intervention $\bar{g}^*(\omega_0)$ as a function of the initial state $\omega_0$, and the pseudo-optimal policy $\tilde{g}(\omega_0)$ that ignores the impact of intervention on firm quality change, treating $\Delta^\omega(\omega;\bar{g})$ as $\Delta^\omega(\omega;0)$ for all $\bar{g}$. In panel (b), we show the percentage decline of social welfare due to using the pseudo-optimal policy instead of the optimal policy.

planner can focus more on the efficiency gain from addressing type-$H$ firms' under-spending.

The mechanism in our model has quantitatively important implications on policy making. The dashed line in Panel A of Figure 5 plots a "pseudo-optimal" $\bar{g}$ that is chosen at $t = 0$ without considering the negative impact of credit intervention on capital quality, $\omega_t$. Specifically, when solving the welfare function given by (27), the planner mistakenly replaces $\Delta^\omega(\omega;\bar{g})$ with $\Delta^\omega(\omega;\bar{g} = 0)$. The resultant "pseudo welfare function", denoted by $\tilde{W}(\omega;\bar{g})$, represents a policy-making criterion that ignores the key mechanism in our paper—credit intervention dampens the cleansing effect of crises—and thus only focuses on the positive impact on capital quantity. Formally, the associated "pseudo-optimal scale of intervention" is given by

$$\tilde{g}^*(\omega_0) = \arg\max_{\bar{g}} \tilde{W}(\omega_0;\bar{g})$$

As shown in Panel A of Figure 5, ignoring the policy impact on capital quality leads to intervention (dashed line) that is almost double the size of optimal intervention (solid line). In Panel B of Figure 5, we plot the percentage decline of welfare from using $\tilde{g}^*(\omega_0)$ rather than the optimal policy

$g^*(\omega_0)$.[38] Ignoring the impact of credit intervention on capital quality translates into a sizeable welfare loss. The welfare cost is largest at the intermediate values of $\omega_0$.

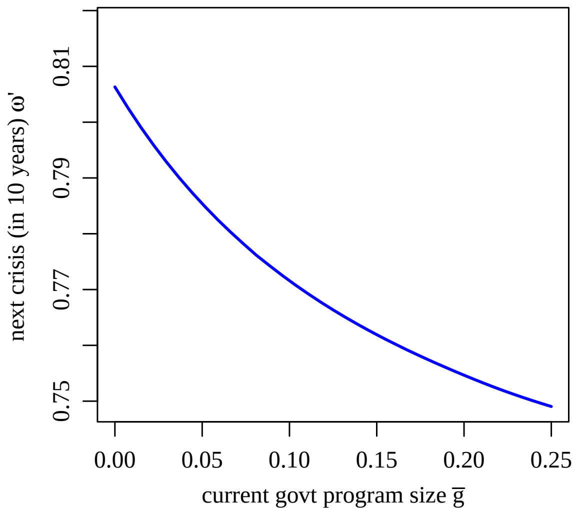
## 3.3 The Slippery Slope of Credit Intervention

The downward bias in firm quality brought by credit intervention generates a slippery slope of intervention. We first show that the impact of credit intervention on $\omega_t$ persists over time. Therefore, intervention in the current crisis leads to a lower $\omega_{t-}$ entering into the next crisis. Accordingly to Proposition 7, a lower $\omega_{t-}$ translates into a greater scale of intervention if the policy maker's goal is to contain the output drop to a certain level. Therefore, in equilibrium, credit intervention in the current crisis begets interventions of greater scales in future crises.

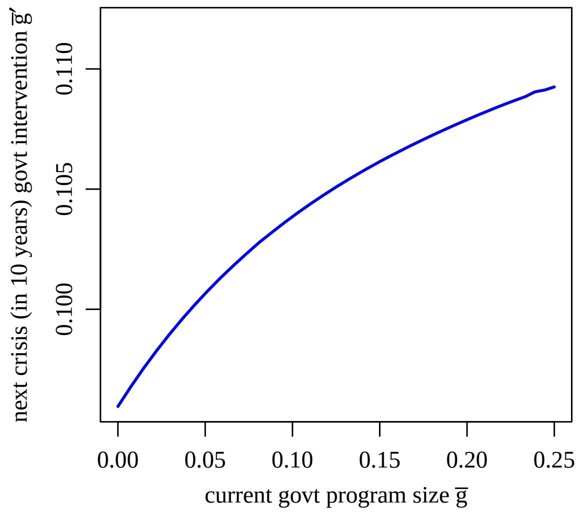
As our focus shifts towards the dynamics of intervention, a key issue to address is agents' expectation of the policy plan. To clearly illustrate the mechanism, we focus on the *forward propagation* of intervention impact: In the current crisis, intervention dampens the cleansing effect, reducing $\omega_t$, and such reduction affects the scale of intervention in future crises. Agents' expectation of the dynamic policy plan confounds the mechanism by introducing a *backward propagation* of intervention impact: agents' current behavior varies with their expectation of intervention in future crises. Agents' expectation enters into the equilibrium conditions only through the capital values, $q^H$ and $q^L$. We shut down this expectation channel by solving $q^H$ and $q^L$ under $\bar{g} = 0$. In Appendix B.1, we consider a fully dynamic model where the government optimally adjusts the scale of intervention in every crisis, and agents, under rational expectation, have perfect knowledge of the policy plan so both the forward and backward propagation are active.

The distortionary effects of credit intervention on firm quality distribution is very persistent. In Panel A of Figure 6, we plot $\omega_t$ ten years ($t = 10$) after the current crisis ($t = 0$) against the intervention size $\bar{g}$ at $t = 0$. As in Section 3.2, the government sets a constant scale of intervention at $t = 0$. We fix the starting firm quality $\omega_0$ at its deterministic steady state in the economy with $\bar{g} = 0$. An increase of credit intervention $\bar{g}$ from 0 to 0.14 (intervention/GDP about 30%, which is our baseline calibration) causes $\omega_t$ to decline by around 0.04 ten years later, which is a significant decline in the percentage of firms being type-$H$. Note that since we shut down agents' expectation of intervention in $q^H$ and $q^L$, the persistent impact of intervention on $\omega_t$ does not come from policy

[38] The curve starts at $\omega_0 = 0$ as it is an absorbing state where the economy is populated by only type-$L$ firms and $\omega_t = \omega_0 = 0$ so credit intervention cannot change $\omega_t$. Note that the other extreme, i.e., $\omega_0 = 1$ where the economy has only type-$H$ firms, is not an absorbing state due to the entry of type-$L$ firms.




(a) Firm quality entering the next crisis



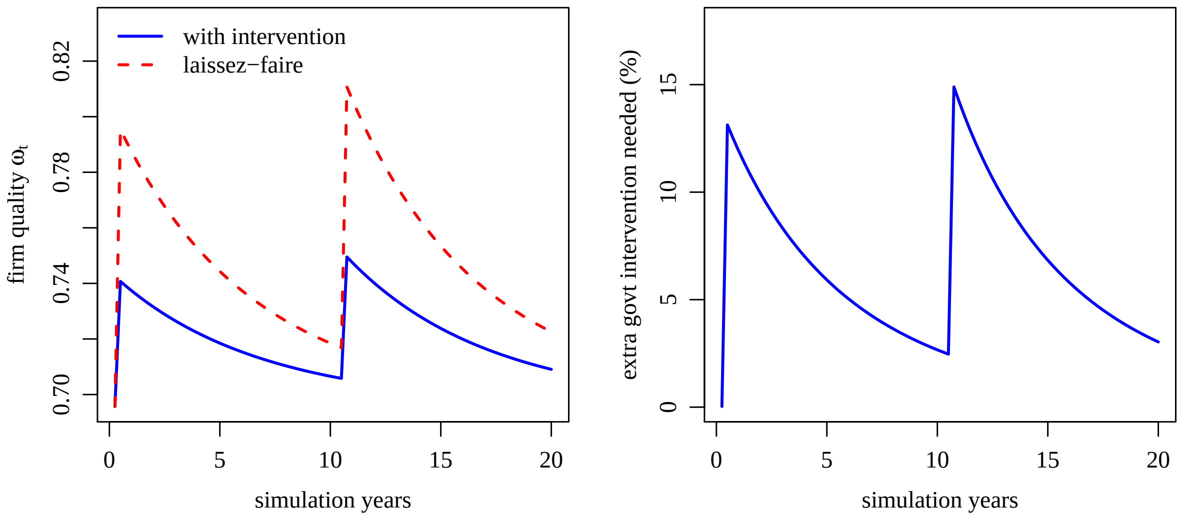

(b) Required intervention in the next crisis

**Figure 6: Intervention pass-through across crises.** We show how $\bar{g}$ in the current crisis affects capital quality entering the next crisis, $\omega'$, and intervention needed, $\bar{g}'$, in the next crisis to contain output drop within $-10\%$. The next crisis happens ten years after the current one. Agents expect no intervention (pass-through is only due to forward propagation). The current crisis happens at $\omega$ equal to the average value of $\omega_t$ in the laissez-faire economy.

distortions in agents' normal-time investments and the drift of $\omega_t$. The persistent impact on $\omega_t$ is purely generated by the reduction of $\omega_t$ in the current crisis at $t = 0$.

Accordingly to Proposition 7, such deterioration of capital quality in the future translates into a greater scale of intervention that is necessary for containing the output drop to a certain level. In Panel B of Figure 6, we consider a crisis that happens ten years from now and plot the necessary scale of intervention against the current scale of intervention. An increase of current intervention $\bar{g}$ from 0 to 0.14 leads to an increase of intervention scale with a pass-through rate of about 7%, i.e., each one dollar of intervention per unit of capital in the current crisis generates 7 cents extra intervention per unit of capital in the next crisis should it happen ten years later. If we take into account the growth of capital stock over the ten-year period, the inter-crisis pass-through rate in the dollar amount is even greater.

We further illustrate the slippery slope of credit intervention with a quarterly simulation of two crises, one in Q1 of the first year and the other in Q1 of the tenth year with $\bar{g}$ equal to 0.14 following our baseline calibration in Section 3.1. In Panel A of Figure 7, we compare the paths of $\omega_t$ in the simulation and in an economy without intervention, both starting from the average




(a) Average capital quality

(b) Additional intervention needed

**Figure 7: The slippery slope of intervention: Simulated paths.** We compare the quarterly simulation of two economies, the laissez-faire economy and the intervened economy without agents expectation of intervention, where one crisis happens in Q1 of the first year and another crisis in Q1 of the tenth year. Both simulations start at the average $\omega_t$ in the laissez-faire economy. In Panel (a), we plot the path of capital quality, $\omega_t$. In Panel (b), for both economies, we calculate the amount of intervention required to contain output drop within $-10\%$ if a crisis happens over the next instant, and then we plot the percentage increase from the laissez-faire economy to the intervened economy.

value of $\omega_t$ in the economy without intervention. After the first crisis, $\omega_t$ jumps upward due to the cleansing effect, but the quality wedge between the laissez-faire economy and intervened economy widens as intervention dampens the cleansing effect. Because we shut down agents' expectation of intervention, firms in both economies have the same normal-time investment rates driven by the same capital values, and $\omega_t$ in both economies converge to the same steady state over time. In spite of the normal-time convergence, the impact of intervention of $\omega_t$ is persistent, evidently shown by the sizable wedge ten years later when the second crisis hits the economy. In the second crisis, the cleansing effect increases $\omega_t$, and the quality wedge widens again.

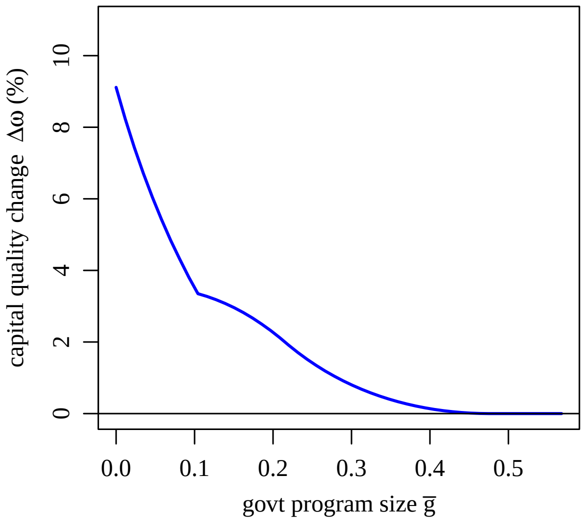
Next, we take as given the simulated paths of $\omega_t$ of the two economies in Panel A of Figure 7, and, at any point in time along the paths, we calculate the necessary scale of intervention were a crisis to happen in the very next instant. In Panel B of Figure 7, we show the extra mount of funding support in the intervened economy relative to that in the laissez-faire economy on percentage terms. Here the government's goal is to contain the output drop within -10%. Because the two economies have the same drift of $\omega_t$, the quality wedge is widest right after a crisis and narrows in normal times. Immediately after the first crisis, if another crisis were to happen, the intervened economy requires about 13% larger funding support from the government than the laissez-faire economy. The wedge shrinks in normal times but still quantitatively significant. For example, five years after the first crisis (i.e., at $t = 5$), if a crisis were to happen over the next instant $dt$, the intervened economy requires a 7% larger funding support from the government than the laissez-faire economy.

In sum, credit intervention biases $\omega_t$ downwards in crises. As a result, the economy enters into future crises with a smaller share of firms being type-$H$ than the laissez-faire benchmark, so the credit support needed to contain output drop is larger. Our model generates a slippery slope of intervention, a trap of policy makers' own making: The past interventions cause the government to spend more should a crisis occur in the future. However, this policy trap can be a necessary evil because by relaxing firms' financial constraints in crises, policy interventions can improve welfare. In Appendix B.1, we show that the same pattern emerges even when the government optimally adjust $\bar{g}$ in a fully dynamic fashion to maximize the social welfare.

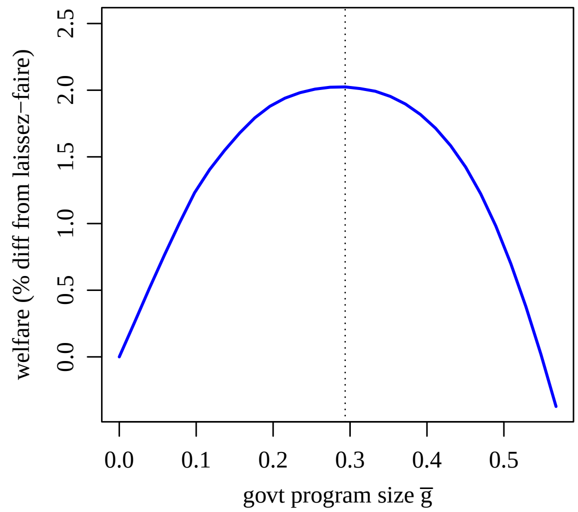
## 3.4 Extension: Alternative Policy Design

The scale of intervention is set proportional to capital stock (i.e., a firm's operational scale in our model), and the government charges market-based interest rates. As previously discussed, this specification follows the policy design in practice. Next, we consider an alternative policy design that improves the welfare. In our model, inefficiency from intervention is from the over-spending by firms that maximize their borrowing and strategically default. These firms do not repay the loans. The firms that actually make repayments, by internalizing the market-based debt costs, do not over-spend. In summary, only firms that efficiently spend the government funding make repayments, and those who abuse it do not repay. Therefore, we consider a new policy: the government eliminates repayment and injects liquidity in the form of subsidy rather than loans.

The new design improves welfare relative to the baseline policy for the following reasons. First, by improving firms' survival probabilities, subsidy reduces the interest rates charged by private-sector creditors. By doing so, it makes strategic default less attractive and repaying loans to private-sector creditors more attractive, enlarging firms' capacity to borrow from private-sector creditors (crowding in "informed liquidity"). This is the extensive margin: the subsidy reduces the number of firms that strategically default and may over-borrow and over-spend. Second, consider




(a) Credit intervention and capital quality




(b) Welfare improvement

**Figure 8: Impact of pure government subsidy: capital quality and welfare.** This figure illustrates how $\bar{g}$ (government intervention is pure subsidy) affects $\Delta\omega_t$ and welfare. The calculation requires the pre-crisis $\omega_{t-}$ which we set to the average value $\bar{\omega}$. For each $\bar{g}$, we solve the model again and calculate the average of simulated $\omega_t$ as $\bar{\omega}(\bar{g})$. In panel B, we show the welfare difference $W(\bar{\omega}(\bar{g});\bar{g})/W(\bar{\omega}(0);0) - 1$ as a function of government intervention $\bar{g}$.

the intensive margin: eliminating repayment helps firms that do not strategically default (note that these firms spend efficiently) but does not change the situation of firms that strategically default. These firms do not repay the government anyway as their goal is to borrow as much as possible to increase survival probability so that their option to hold up creditors becomes in-the-money.

What we propose seems the opposite to the famous Bagehot's Dictum—central banks should lend freely at high rates in crises. In Bagehot (1897), the condition behind this policy recommendation is that central banks only lend to solvent firms and only lend against good collateral. This condition requires central banks to be informed about firms' solvency and collateral quality. In our paper, we study liquidity support provided by central banks and governments in general, and the starting point of our analysis is the lack of differentiating among firms of different qualities. Therefore, our model leads to a policy recommendation that differs from Bagehot's. We acknowledge that this subsidy-based liquidity injection improves welfare relative to the baseline policy in our specific setting that may not reflect fully the complexity of realistic policy-making environments.

In Appendix D.4, we discuss details on the model solution. In Figure 8, we illustrate how government intervention in the form of subsidy affects the change of firm quality in a crisis and welfare. In panel A, we find that increasing the scale of intervention still lowers firm quality in a crisis. Given $\bar{g}$, the impact seems stronger than that in the baseline model (see Figure 4) because, without repayment, subsidy reduces more take-up than loans. There is a kink point around $\bar{g} = 0.1$, above which the subsidy is so high that all firms choose not to strategically default on private-sector creditors. When firms can rely more on government subsidy, they borrow less from private-sector creditors so the benefit of strategic default diminishes.[39] Below this kink point, some firms still strategically default on private-sector creditors, and the inefficiency of over-borrowing exists; given that type-$L$ firms have stronger incentive to over-borrow and strategically default, intervention dampens the cleansing effect of crises more strongly below the kink point than above the point.

In panel B of Figure 8, we find that there is a greater increase of welfare for a wider range of $\bar{g}$ than what Panel B of Figure 4 shows for the main model. Moreover, the optimal intervention scale, indicated by the peak of welfare curve, is larger. Therefore, government intervention in the form of subsidy is more efficient than loans in spite of the fact that intervention still distorts firm quality dynamics. In Appendix D.4, we provide more details and discussion on the model solution and show that intervention in the form of subsidy also features a slippery slope that is quantitatively similar to that of intervention in the form of loans.

# 4 Conclusion

To analyze the long-term consequences of credit intervention in crises, we develop a model of firm quality dynamics and highlight a trade-off between quantity and quality in determining the scale of intervention. Crises exhibit a cleansing effect: Firms with high productivity want to spend more on surviving a crisis, and they can spend more than firms with low productivity because their financing capacity is larger. Credit intervention relaxes the financial constraint for all firms and preserves the total production capacity in the economy. However, by benefiting firms with low productivity more, credit intervention dampens the cleansing effect of crises. Our model generates a slippery slope of intervention. As the current intervention biases downward the firm quality distribution, the economy enters the next crisis with lower total productivity, and an intervention of a greater scale becomes necessary. Larger interventions lead to stronger distortions, which in turn call for even larger interventions in the future. However, we show that when carefully designed,

[39] Firm owners' value from strategic default, $\beta$, is below capital value, so if subsidy leads to less borrowing from (and repayment to) private-sector creditors, firms owners choose to keep capital (control right) rather than go bankrupt.

credit intervention improves welfare relative to the laissez-faire benchmark.

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# Internet Appendix to
## "Firm Quality Dynamics and the Slippery Slope of Credit Intervention"

Wenhao Li    Ye Li

# A Proofs

**Proof of Lemma 1.** See proof in the main text.

**Proof of Lemma 2.** Using the creditors' break-even condition (13) to substitute out the interest rate in the value under debt repayment, we obtain

$$q_t^j - \left(1 + r_t^j(\zeta, x_t^j(\zeta))\right) x_t^j(\zeta) = q_t^j - \frac{x_t^j(\zeta)}{F(x_t^j(\zeta) + \zeta)}. \tag{A1}$$

Consider two cases. First, the private-market funding constraint binds, so $x_t^j(\zeta) = \bar{d}$. We have

$$q_t^j - \left(1 + r_t^j(\zeta, x_t^j(\zeta))\right) x_t^j(\zeta) = q_t^j - \frac{\bar{d}}{F(\bar{d} + \zeta)}, \tag{A2}$$

which increases in $\zeta$ because $F(\cdot)$ is an increasing function. Second, the private-market funding constraint does not bind, so $x_t^j(\zeta) = \bar{\zeta}_t^j - \zeta$. We have

$$q_t^j - \left(1 + r_t^j(\zeta, x_t^j(\zeta))\right) x_t^j(\zeta) = q_t^j - \frac{\bar{\zeta}_t^j - \zeta}{F(\bar{\zeta}_t^j)}, \tag{A3}$$

which increases in $\zeta$. The value under debt repayment is monotonic in $\zeta$, so there exists a unique $\underline{\zeta}_t^j$ where the value under debt repayment as a function of $\zeta$ is equal to $\beta$.

Note that so far, we have characterized the value under debt repayment for the whole space of $\zeta$, i.e., the function $q_t^j - \left(1 + r_t^j(\zeta, x_t^j(\zeta))\right) x_t^j(\zeta)$ of $\zeta$ on the domain $\zeta \in [0, +\infty]$, and we show that this function is equal to $\beta$ at a unique value $\zeta = \underline{\zeta}_t^j$. Next, we prove that this $\underline{\zeta}_t^j$ is indeed the default threshold; that is, if a firm survives, it will default if $\zeta < \underline{\zeta}_t^j$ and repay its debt if $\zeta \geq \underline{\zeta}_t^j$. First, consider $\zeta \geq \underline{\zeta}_t^j$. If $\underline{\zeta}_t^j$ is the default threshold, the creditors know it and know that the firm with $\zeta \geq \underline{\zeta}_t^j$ will not default conditional on survival. Therefore, when the firm borrows, it faces an interest rate function, $r_t^j(\zeta, x)$ given by (13) and borrow $x_t^j(\zeta)$ given by (16). After survival, the firm will not default because the value from repaying its debt, $q_t^j - \left(1 + r_t^j(\zeta, x_t^j(\zeta))\right) x_t^j(\zeta)$ is higher than $\beta$. Next, consider $\zeta < \underline{\zeta}_t^j$. We prove that a firm with $\zeta < \underline{\zeta}_t^j$ will default after survival by contradiction. Suppose that this firm does not default (i.e., $\zeta < \underline{\zeta}_t^j$ is not the criterion for default). The creditors know that this firm does not default and offer an interest rate function, $r_t^j(\zeta, x)$ given by (13). Since the firm will not default conditional on survival, it will borrow $x_t^j(\zeta)$ given by (16) before survival. After survival, its value is $q_t^j - \left(1 + r_t^j(\zeta, x_t^j(\zeta))\right) x_t^j(\zeta)$, which is below $\beta$ because $\zeta < \underline{\zeta}_t^j$. This is a contradiction. Therefore, for firms with $\zeta < \underline{\zeta}_t^j$, strategic default happens. And, as previously shown, for firms with $\zeta \geq \underline{\zeta}_t^j$, debts are repaid. Therefore, we first prove that $q_t^j - \left(1 + r_t^j(\zeta, x_t^j(\zeta))\right) x_t^j(\zeta)$ is an increasing function of $\zeta$ that equals $\beta$ at $\zeta = \underline{\zeta}_t^j$, and then we prove that $\underline{\zeta}_t^j$ is threshold with $\zeta$ below which a firm strategically defaults.

Next, we show that given $\zeta$, the value under debt repayment increases in $q_t^j$ This is obvious when the private-market funding constraint binds, i.e., $x_t^j(\zeta) = \bar{d}$, and the value under debt repayment is $q_t^j - \frac{\bar{d}}{F(\bar{d}+\zeta)}$. When the private-market funding constraint does not bind, i.e., $x_t^j(\zeta) = \bar{\zeta}_t^j - \zeta$, we calculate the derivative of the value under debt repayment with respect to $q_t^j$:

$$\frac{\partial \left( q_t^j - \frac{\bar{\zeta}_t^j - \zeta}{F(\bar{\zeta}_t^j)} \right)}{\partial q_t^j} = 1 - \frac{F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)(\bar{\zeta}_t^j - \zeta)}{F(\bar{\zeta}_t^j)^2} \frac{d\bar{\zeta}_t^j}{dq_t^j} \tag{A4}$$

Differentiate (14) with respect to $q_t^j$, we obtain

$$F''(\bar{\zeta}_t^j) q_t^j \frac{d\bar{\zeta}_t^j}{dq_t^j} + F'(\bar{\zeta}_t^j) = 0\,, \tag{A5}$$

and rearranging the equation, we solve

$$\frac{d\bar{\zeta}_t^j}{dq_t^j} = -\frac{F'(\bar{\zeta}_t^j)}{F''(\bar{\zeta}_t^j) q_t^j}\,. \tag{A6}$$

Using this expression to substitute out $\frac{d\bar{\zeta}_t^j}{dq_t^j}$ in (A4), we obtain

$$\begin{aligned}\frac{\partial\left(q_t^j - \frac{\bar{\zeta}_t^j - \zeta}{F(\bar{\zeta}_t^j)}\right)}{\partial q_t^j} =& 1 - \frac{F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)(\bar{\zeta}_t^j - \zeta)}{F(\bar{\zeta}_t^j)^2}\frac{d\bar{\zeta}_t^j}{dq_t^j} = 1 + \frac{F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)(\bar{\zeta}_t^j - \zeta)}{F(\bar{\zeta}_t^j)^2}\frac{F'(\bar{\zeta}_t^j)}{F''(\bar{\zeta}_t^j)q_t^j} \\ =& 1 + \frac{F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)(\bar{\zeta}_t^j - \zeta)}{\left(F(\bar{\zeta}_t^j)q_t^j\right)^2 F''(\bar{\zeta}_t^j)}\end{aligned} \tag{A7}$$

Because $F(\cdot)$ is concave and $F(0) = 0$, we have

$$F(\bar{\zeta}_t^j) = F(\bar{\zeta}_t^j) - F(0) \geq F'(\bar{\zeta}_t^j)\bar{\zeta}_t^j$$

Therefore, we obtain

$$F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)(\bar{\zeta}_t^j - \zeta) = F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)\bar{\zeta}_t^j + F'(\bar{\zeta}_t^j)\zeta > 0\,, \tag{A8}$$

for all $\zeta > 0$ because $F'(\cdot) > 0$. Using this result, we have shown that

$$\frac{\partial\left(q_t^j - \frac{\bar{\zeta}_t^j - \zeta}{F(\bar{\zeta}_t^j)}\right)}{\partial q_t^j} = 1 + \frac{F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)(\bar{\zeta}_t^j - \zeta)}{\left(F(\bar{\zeta}_t^j)q_t^j\right)^2 F''(\bar{\zeta}_t^j)} > 1 > 0\,. \tag{A9}$$

In sum, we have shown that given $\zeta$, the value under debt repayment increases in $q_t^j$ whether the private-market funding constraint binds or not. Therefore, for a higher $q_t^j$, the value under debt repayment is higher for all $\zeta$. Since the value under debt repayment is increasing in $\zeta$, this implies that $\underline{\zeta}_t^j$ is lower when $q_t^j$ is higher, i.e., the value under debt repayment is equal to the value under strategic default, $\beta$ at a lower value of $\zeta$. Thus, $\underline{\zeta}_t^j$ is a decreasing function of $q_t^j$.

**Proof of Lemma 3 and 4.** First, we show that the following expression is monotonic in $d$ for any $\zeta > 0$,

$$\hat{\pi}^j(d) = F(d + \zeta + \bar{g})(q_t^j - \beta) - d$$

We note that the derivative over $d$ is

$$\hat{\pi}^j(d)' = F'(d + \zeta + \bar{g})(q_t^j - \beta) - 1$$

Replacing $q_t^j - \beta$ with equation (18), the above derivative can be simplified as

$$\hat{\pi}^j(d)' = \frac{F'(d+\zeta+\bar{g})d}{F(d+\zeta+\bar{g})} - 1$$

By strict concavity of $F(\cdot)$, we have

$$F(d+\zeta+\bar{g}) > F'(d+\zeta+\bar{g})(d+\zeta) > F'(d+\zeta+\bar{g})d$$

Therefore, we obtain $\hat{\pi}^j(d)' < 0$ so that $\hat{\pi}^j(d)$ is a decreasing function. Since for any $\zeta > 0$, $\hat{\pi}^j(0) > 0$ while $\hat{\pi}^j(\infty) = -\infty$, a solution exists and it is unique.

Next, since $\hat{\pi}^j(d)$ increases in $\zeta$, $\hat{d}^j(\zeta)$ as the solution to $\hat{\pi}^j(d) = 0$ increases in $\zeta$. Similarly, $\hat{d}^j(\zeta)$ also increases in $q_t^j$, which implies $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$, and it increases in $\bar{g}$ (Lemma 4).

Next, we prove that $\hat{d}^j(\zeta)$ is strictly concave in $\zeta$. Differentiating the the creditors' break-even condition (18), we obtain:

$$F'(\hat{d}^j(\zeta)+\zeta)(\hat{d}^{j\prime}(\zeta)+1)(q_t^j-\beta) = \hat{d}^{j\prime}(\zeta)\,. \tag{A10}$$

Rearranging the equation, we have

$$\hat{d}^{j\prime}(\zeta) = \left(\frac{1}{F'(\hat{d}^j(\zeta)+\zeta)(q_t^j-\beta)} - 1\right)^{-1}. \tag{A11}$$

From this expression, we can see that $\hat{d}^{j\prime}(\zeta)$ decreases in $\zeta$, i.e., $\hat{d}^j(\zeta)$ is strictly concave in $\zeta$.

Finally, we show the impact of government intervention on the interest rate. The interest-rate determination equation is

$$F(x_t^j+\zeta)(1+r_t^j) = 1$$

Since $x_t^j$ increases with government intervention $\bar{g}$, the interest rate $r_t^j$ decreases with higher $\bar{g}$. This decreasing relationship is strict when $\zeta_t^j - \zeta > \bar{d} + \bar{g}$.

**Proof of Proposition 1** Summarizing Lemma 1, 2, and 3, we obtain $\bar{\zeta}_t^H > \bar{\zeta}_t^L$, $\underline{\zeta}_t^H < \underline{\zeta}_t^L$, and $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$ under $q_t^H > q_t^L$.

Next, property (1) and (2) are direct results of equation (19).

**Proof of Proposition 2** Results directly follow the comparison between the first-best solution in equation (15) and the optimal financing in equation (19).

**Proof of Corollary 1** Let's consider the case of $\bar{g} \geq \bar{\zeta}_t^H$. In this case, government credit alone can satisfy the capacity need by the first-best solution. We find that under this scenario, by further expanding government intervention $\bar{g}$, the efficiency gain in Proposition 2 is not affected, but the efficiency loss in Proposition 2 is further increased. When $\bar{g} \to \infty$, the survival probability $F(\bar{d} + \bar{g}) \to 1$, so the efficiency change is approximately $F(\bar{d} + \bar{g})q_t^j - (\bar{d} + \bar{g}) \to -\infty$. Consequently, the efficiency loss dominates the efficiency gain for a sufficiently large scale of government intervention.

**Proof of Proposition 3** In the capital price equation (21), the optimal borrowing decision $x_t^j(\zeta)$ and interest rate $r_t^j$ are both time-invariant functions of $\zeta$ and $q_t^j$, and the optimal investment $\iota_t^j$ is a time-invariant function of $q_t^j$. When $q_t^j$ is a constant, the variables above are constant, which in turn imply a constant $q_t^j$ according to (21). Therefore, we have shown the existence of a stationary equilibrium.

Next, we prove uniqueness. We omit the subscript $t$ since all variables are time-invariant in the stationary equilibrium. Rewriting equation (21), we obtain

$$0 = A^j - \phi(\iota^j) + \iota^j q^j - (\delta + r)q^j + \lambda \left( \int_0^\infty F(x^j(\zeta) + \zeta) \max\{q^j - (1 + r^j)x^j(\zeta), \beta\} dH(\zeta) - q^j \right)$$

With the interest-rate determination in equation (13), we can simplify the problem as

$$0 = A^j - \phi(\iota^j) + \iota^j q^j - (\delta + r)q^j + \lambda \left( \int_0^\infty \max\{F(x^j(\zeta) + \zeta)q^j - x^j(\zeta), \beta F(x_t^j(\zeta) + \zeta)\} dH(\zeta) - q^j \right) \tag{A12}$$

Note that, inside the integral in the last term, $\max\{F(x_t^j(\zeta) + \zeta)q^j - x^j(\zeta), \beta F(x_t^j(\zeta) + \zeta)\}$ is the objective function of a type-$j$ firm with a realized $\zeta$. When the funding constraint binds, i.e., $x_t^j(\zeta) = \bar{g} + \bar{d}$, the first term in the max operator increases in $q^j$ and the second term is non-decreasing in $q^j$. When the funding constraint does not bind, the envelope theorem implies that $\max\{F(x_t^j(\zeta) + \zeta)q^j - x^j(\zeta), \beta F(x_t^j(\zeta) + \zeta)\}$ is (weakly) increasing in $q^j$.

Uniqueness will follow if we can prove the monotonicity over $q^j$ in the right-hand side of the above equation. First, we analyze $-\phi(\iota_t^j) + \iota^j q^j - (\delta + r)q^j$. Note that the investment cost is

$$\phi(\iota^j) = \frac{q^j - 1}{\theta} + \frac{\theta}{2}\left(\frac{q^j - 1}{\theta}\right)^2 = \frac{1}{2\theta}(q^j)^2 - \frac{1}{2\theta}$$

Thus,

$$-\phi(\iota^j) + \iota^j q^j - (\delta + r)q^j = \frac{1}{2\theta}(q^j)^2 - (r + \delta + \frac{1}{\theta})q^j + \frac{1}{2\theta}$$

Under the condition on investment $i^j \leq r + \delta$ (i.e., investment rate lower than the total discount rate on capital), we get

$$q^j \leq 1 + \theta(r + \delta)$$

which implies that $-\phi(\iota^j) + \iota^j q^j - (\delta + r)q^j$ strictly decreases in $q^j$.

Next, we discuss the crisis-spending component in the large bracket. For convenience of discussion, we rewrite the integrand as

$$\begin{aligned}
&\max\{\left(F(x_t^j(\zeta) + \zeta) - 1\right) q^j - x^j(\zeta), \beta F(x_t^j(\zeta) + \zeta) - q^j\} \\
=&\mathbf{1}_{\zeta \geq \underline{\zeta}^j}\left(\max_{x \leq \bar{d} + \bar{g}}\{F(x + \zeta)q^j - x - q^j\}\right) + \mathbf{1}_{\zeta < \underline{\zeta}^j}\left(\beta F(\min\{\hat{d}^j(\zeta), \bar{d}\} + \bar{g} + \zeta) - q^j\right)
\end{aligned}$$

First, consider the region of $\zeta \geq \underline{\zeta}^j$. For each $x$ and $\zeta$, the expression $F(x + \zeta)q^j - x - q^j$ is a decreasing function of $q^j$. As a result, $\max_{x \leq \bar{d} + \bar{g}}\{F(x + \zeta)q^j - x - q^j\}$ decreases in $q^j$.

Second, consider the region of $\zeta$ where the firm strategically defaults, i.e., $\zeta < \underline{\zeta}^j$, and the private-market debt limit is above the amount of available funding, i.e., $\hat{d}^j(\zeta) \geq \bar{d}$. The firm's value in crises, $\beta F(\min\{\hat{d}^j(\zeta), \bar{d}\} + \bar{g} + \zeta) - q^j = \beta F(\bar{d} + \bar{g} + \zeta) - q^j$ decreases in $q^j$.

Third, consider the region of $\zeta$ where the firm strategically defaults, i.e., $\zeta < \underline{\zeta}^j$, the funding constraint does not bind, i.e., $\hat{d}^j(\zeta) < \bar{d}$, and the firm over-spends. We calculate the derivative of $\hat{d}^j$ over $q^j$. Taking derivative over $q^j$ on equation (18),

$$F'(\hat{d}^j(\zeta) + \bar{g} + \zeta)(q^j - \beta)\frac{\partial \hat{d}^j(\zeta)}{\partial q^j} + F(\hat{d}^j(\zeta) + \bar{g} + \zeta) = \frac{\partial \hat{d}^j(\zeta)}{\partial q^j}$$

$$\Rightarrow \frac{\partial \hat{d}^j(\zeta)}{\partial q^j} = \frac{F(\hat{d}^j(\zeta) + \bar{g} + \zeta)}{1 - F'(\hat{d}^j(\zeta) + \bar{g} + \zeta)(q^j - \beta)}$$

Thus, we have

$$
\begin{aligned}
&\frac{\partial\left(\beta F(\hat{d}_t^j(\zeta)+\bar{g}+\zeta)-q^j\right)}{\partial q^j}\\
=&\beta F'(\hat{d}_t^j(\zeta)+\bar{g}+\zeta)\frac{\partial \hat{d}_t^j(\zeta)}{\partial q^j}-1\\
=&\frac{\beta F'(\hat{d}_t^j(\zeta)+\bar{g}+\zeta)F(\hat{d}^j(\zeta)+\bar{g}+\zeta)}{1-F'(\hat{d}^j(\zeta)+\bar{g}+\zeta)(q^j-\beta)}-1\\
=&\frac{\beta F'(\hat{d}_t^j(\zeta)+\bar{g}+\zeta)F(\hat{d}^j(\zeta)+\bar{g}+\zeta)-1+F'(\hat{d}^j(\zeta)+\bar{g}+\zeta)(q^j-\beta)}{1-F'(\hat{d}^j(\zeta)+\bar{g}+\zeta)(q^j-\beta)}\\
=&\frac{F'(\hat{d}_t^j(\zeta)+\bar{g}+\zeta)\beta\left(F(\hat{d}^j(\zeta)+\bar{g}+\zeta)-1\right)+F'(\hat{d}^j(\zeta)+\bar{g}+\zeta)q^j-1}{1-F'(\hat{d}^j(\zeta)+\bar{g}+\zeta)(q^j-\beta)}
\end{aligned}
$$

If we have

$$F'(\hat{d}^j(\zeta)+\bar{g}+\zeta)q^j-1<0 \tag{A13}$$

then the derivative is negative because the first item in the numerator is negative ($F'(\cdot)>0$ and $F(\cdot)<1$), the second item in the numerator is negative implied by (A13), and the denominator is positive also implied by (A13). Note that inequality (A13) is equivalent to

$$\hat{d}^j(\zeta)+\bar{g}+\zeta>\bar{\zeta}^j$$

Therefore in the region of $\zeta$ where over-spending happens, we have $\frac{\partial\left(\beta F(\hat{d}_t^j(\zeta)+\bar{g}+\zeta)-q^j\right)}{\partial q^j}<0$.

Finally, consider the region of $\zeta$ where the firm strategically defaults, i.e., $\zeta<\underline{\zeta}^j$, the funding constraint does not bind, i.e., $\hat{d}^j(\zeta)<\bar{d}$, and the firm under-spends. In this region, we impose a sufficient parametric condition that guarantees the firm's value in crises decreasing in $q^j$. From the previous calculation, we have

$$\frac{d[\beta F(\hat{d}^j(\zeta)+\bar{g}+\zeta)]}{dq^j}=\frac{\beta F'(\hat{d}^j(\zeta)+\bar{g}+\zeta)F(\hat{d}^j(\zeta)+\bar{g}+\zeta)}{1-F'(\hat{d}^j(\zeta)+\bar{g}+\zeta)(q^j-\beta)} \tag{A14}$$

Given $F(z)=1-e^{\lambda_F z}$, i.e., an exponential distribution, we have $F'(z)=\lambda_F(1-F(z))$ and obtain

$$\frac{d[\beta F(\hat{d}^j(\zeta)+\bar{g}+\zeta)]}{dq^j}=\frac{\beta\lambda_F(1-F(\hat{d}^j_G(\zeta)+\bar{g}+\zeta))F(\hat{d}^j_G(\zeta)+\bar{g}+\zeta)}{1-F'(\hat{d}^j(\zeta)+\bar{g}+\zeta)(q^j-\beta)} \tag{A15}$$

The numerator contains a quadratic form of $F(\hat{d}^j_G(\zeta)+\bar{g}+\zeta)\in(0,\,1)$, i.e., $(1-F(\hat{d}^j_G(\zeta)+\bar{g}+\zeta))F(\hat{d}^j_G(\zeta)+\bar{g}+\zeta)$, which has a maximum equal to 1/4, obtained at $F(\hat{d}^j_G(\zeta)+\bar{g}+\zeta)=1/2$. Therefore,

$$\frac{d[\beta F(\hat{d}^j(\zeta)+\bar{g}+\zeta)]}{dq^j}\leq\frac{\beta\lambda_F}{4[1-F'(\hat{d}^j(\zeta)+\bar{g}+\zeta)(q^j-\beta)]} \tag{A16}$$

Moreover, using $F'(\hat{d}^j(\zeta)+\bar{g}+\zeta)=\lambda_F e^{-\lambda_F(\hat{d}^j(\zeta)+\bar{g}+\zeta)}\leq\lambda_F$, we have

$$\frac{d[\beta F(\hat{d}^j(\zeta)+\bar{g}+\zeta)]}{dq^j}\leq\frac{\beta\lambda_F}{4[1-\lambda_F(q^j-\beta)]} \tag{A17}$$

We impose the parametric restriction

$$\frac{d[\beta F(\hat{d}^j(\zeta)+\bar{g}+\zeta)]}{dq^j}\leq\frac{\beta\lambda_F}{4[1-\lambda_F(q^j-\beta)]}<1\,. \tag{A18}$$

This guarantees that $\beta F(x^j_t(\zeta)+\zeta)=\beta F(\min\{\hat{d}^j(\zeta),\bar{d}\}+\bar{g}+\zeta)$, has a derivative with respect to $q^j$ that is smaller than one, i.e., the firm value in crises, $\beta F(x^j_t(\zeta)+\zeta)-q^j$, decreases in $q^j$.

We have shown hat the integrand in the crisis component decreases in $q^j$ for any $\zeta$. Therefore, the integral decreases in $q^j$. In sum, the right side of (A12) decreases in $q^j$, and therefore, the solution of $q^j$ is unique. Moreover, $A^H>A^L$ implies $q^H>q^L$.

**Proof of Proposition 4** In the first-best economy describe by equation (15), we have

$$\int F(x^H_t(\zeta)+\zeta)dH(\zeta)>\int F(x^L_t(\zeta)+\zeta)dH(\zeta) \tag{A19}$$

Therefore, $\kappa^H_t>\kappa^L_t$, which leads to $\Delta^\omega_t(\omega_{t-})>0$ according to equation (24).

**Proof of Corollary 2** First, we consider introducing only a tight constraint ($\bar{d}\to 0$) to the first-best economy (without funding constraint, $\bar{d}=\infty$, and no government intervention yet so $\bar{g}=0$).

In this case, $\beta = 0$, so there is no strategic default and the optimal financing is reduced to

$$x_t^j(\zeta) = \min\{(\bar{\zeta}_t^j - \zeta)^+, \bar{d} + \bar{g}\} \tag{A20}$$

We need to prove that

$$\Delta_t^\omega(\bar{d} = \infty; \bar{g} = 0; \beta = 0) > \Delta_t^\omega(\bar{d} \to 0; \bar{g} = 0; \beta = 0)$$

Using the definition of $\kappa_t^j$ in (23), in the case of $\bar{d} = \infty$ and $\beta = 0$, we get $\kappa_t^H > \kappa_t^L$, so there is a cleansing effect,

$$\Delta_t^\omega(\bar{d} = \infty; \bar{g} = 0; \beta = 0) > 0$$

In the case of $\bar{d} \to 0$, $\bar{g} = 0$, and $\beta = 0$, we get $\kappa_t^H = \kappa_t^L$, so there is no cleansing effect,

$$\Delta_t^\omega(\bar{d} \to 0; \bar{g} = 0; \beta = 0) = 0$$

Consequently, introducing a tight financial constraint to private-sector funding supply weakens the cleansing effect.

Next, we need to prove that the marginal effect of government financing is positive on cleansing, i.e.,

$$\frac{\partial \Delta_t^\omega}{\partial \bar{g}}|_{\bar{d} \to 0, \bar{g} = 0, \beta = 0} > 0 \tag{A21}$$

According to the definition of $\Delta_t^\omega$ in (24), $\partial \Delta_t^\omega / \partial \bar{g} > 0$ is equivalent to

$$\frac{\partial(\kappa_t^L/\kappa_t^H)}{\partial \bar{g}} < 0 \Leftrightarrow \frac{\partial \kappa_t^L}{\partial \bar{g}}/\kappa_t^L < \frac{\partial \kappa_t^H}{\partial \bar{g}}/\kappa_t^H. \tag{A22}$$

Using (A20) and the definition of $\kappa_t^j$ in (23), we get

$$\frac{\partial \kappa_t^j}{\partial \bar{g}} = \int_{(\bar{\zeta}_t^j - \zeta)^+ \geq \bar{d} + \bar{g}} F'(\bar{d} + \bar{g} + \zeta) dH(\zeta)$$

Therefore, for $\bar{d} + \bar{g} < \bar{\zeta}_t^H$, we have

$$\frac{\partial \kappa_t^L}{\partial \bar{g}} < \frac{\partial \kappa_t^H}{\partial \bar{g}}$$

Since we consider the expansion of government intervention at $\bar{d} \to 0$ and $\bar{g} = 0$, $\kappa_t^L = \kappa_t^H$. As a result, we have proven

$$\frac{\partial \kappa_t^L}{\partial \bar{g}}/\kappa_t^L < \frac{\partial \kappa_t^H}{\partial \bar{g}}/\kappa_t^H,$$

which according to (A22) leads to (A21). In summary, government financing strengthens the cleansing effect in this special case of $\beta = 0$ and tight financial constraint.

**Proof of Corollary 3** Consider $\zeta \in \mathcal{U}$ for $j \in \{H, L\}$ such that $\hat{d}^j(\zeta) < \bar{d}$ and $\zeta < \underline{\zeta}_t^j$. Define $\hat{H}(\zeta) = H(\zeta)$ if $\zeta \in \mathcal{U}$ and $\hat{H}(\zeta) = 0$ if $\zeta \notin \mathcal{U}$. Among these type-$j$ firms that strategically default, the measure of surviving firms is given by

$$\hat{\kappa}_t^j = \int F(\hat{d}_t^j + \bar{g} + \zeta) d\hat{H}(\zeta)$$

Under $\hat{d}_t^H > \hat{d}_t^L$, we have $\hat{\kappa}_t^H > \hat{\kappa}_t^L$. Therefore, the endogenous debt limit contributes to the cleansing effect of crises.

Next, we consider how credit intervention affects this channel of cleansing effect. Specifically, we prove among firms that strategically default, credit intervention reduces the share of surviving firms that are type-$H$, i.e.,

$$\frac{\partial(\hat{\kappa}_t^H/(\hat{\kappa}_t^H + \hat{\kappa}_t^L))}{\partial \bar{g}} < 0 \Leftrightarrow \frac{\partial \hat{\kappa}_t^L}{\partial \bar{g}}/\hat{\kappa}_t^L > \frac{\partial \hat{\kappa}_t^H}{\partial \bar{g}}/\hat{\kappa}_t^H \Leftrightarrow \frac{\partial(\hat{\kappa}_t^L/\hat{\kappa}_t^H)}{\partial \bar{g}} > 0. \tag{A23}$$

We note that

$$\frac{\partial \hat{\kappa}_t^j}{\partial \bar{g}}/\hat{\kappa}_t^j = \frac{\int F'(\hat{d}_t^j + \bar{g} + \zeta) d\hat{H}(\zeta)}{\int F(\hat{d}_t^j + \bar{g} + \zeta) d\hat{H}(\zeta)}.$$

Define

$$\mathcal{L}(u) = \frac{\int F'(u + \zeta) d\hat{H}(\zeta)}{\int F(u + \zeta) d\hat{H}(\zeta)}.$$

Then the derivative of $\mathcal{L}(u)$ is

$$\mathcal{L}'(u) = \frac{\left(\int F(u+\zeta) d\hat{H}(\zeta)\right)\left(\int F''(u+\zeta) d\hat{H}(\zeta)\right) - \left(\int F'(u+\zeta) d\hat{H}(\zeta)\right)^2}{\left(\int F(u+\zeta) d\hat{H}(\zeta)\right)^2}.$$

Under $F''(\cdot) < 0$, we have $\mathcal{L}'(u) < 0$, which implies $\frac{\partial \hat{\kappa}_t^L}{\partial \bar{g}}/\hat{\kappa}_t^L > \frac{\partial \hat{\kappa}_t^H}{\partial \bar{g}}/\hat{\kappa}_t^H$.

**Proof of Proposition 5** First, by definition of $\kappa_t^j$ in (23) and $F(x) < 1$ for $x < \infty$, we obtain $\kappa_t^j < 1$ for $j \in \{L, H\}$.

Next, under Assumption 1, we are able to provide a stronger statement that firm quality jumps up in a crisis, $\Delta_t^\omega > 0$. According to (24), $\Delta_t^\omega > 0$ is equivalent to $\kappa_t^H > \kappa_t^L$, which by definition in (23) is equivalent to

$$\underbrace{\int F(x_t^H(\zeta) + \zeta) dH(\zeta)}_{\kappa_t^H} > \underbrace{\int F(x_t^L(\zeta) + \zeta) dH(\zeta)}_{\kappa_t^L}$$

Under Assumption 1 and $\bar{g} = 0$, the optimal financing choices of H and L types are

$$x_t^H(\zeta) = \min\{(\bar{\zeta}_t^H - \zeta)^+, \bar{d} + \bar{g}\} \tag{A24}$$

$$x_t^L(\zeta) = \mathbf{1}_{\zeta \geq \underline{\zeta}_t^L} \min\{(\bar{\zeta}_t^L - \zeta)^+, \bar{d} + \bar{g}\} + \mathbf{1}_{\zeta < \underline{\zeta}_t^L} \left( \min\{\hat{d}_t^L(\zeta), \bar{d}\} + \bar{g} \right) \tag{A25}$$

For $\zeta \geq \underline{\zeta}_t^L$, we have

$$x_t^H(\zeta) = \min\{(\bar{\zeta}_t^H - \zeta)^+, \bar{d} + \bar{g}\} \geq \min\{(\bar{\zeta}_t^L - \zeta)^+, \bar{d} + \bar{g}\} = x_t^L(\zeta)$$

where the inequality is strict for $\zeta \geq \max\{\bar{\zeta}_t^H - (\bar{d} + \bar{g}), \underline{\zeta}_t^L\}$.

For $\zeta < \underline{\zeta}_t^L$, we need to compare $\bar{\zeta}_t^H - \zeta$ with $\hat{d}_t^L(\zeta) + \bar{g}$. In this region, according to the monotonicity of $\hat{d}_t^L(\zeta)$ in Lemma 3, we get $\hat{d}_t^L(\zeta) \leq \hat{d}_t^L(\underline{\zeta}_t^L)$. Therefore,

$$\hat{d}_t^L(\zeta) + \zeta \leq \hat{d}_t^L(\underline{\zeta}_t^L) + \underline{\zeta}_t^L < \bar{\zeta}_t^H - \bar{g},$$

which implies that

$$\bar{\zeta}_t^H - \zeta > \hat{d}_t^L(\zeta) + \bar{g}$$

Consequently,

$$x_t^H(\zeta) = \min\{(\bar{\zeta}_t^H - \zeta)^+, \bar{d} + \bar{g}\} \geq \min\{\hat{d}_t^L(\zeta), \bar{d}\} + \bar{g} = x_t^L(\zeta)$$

In summary, for any $\zeta$, we have $x_t^H(\zeta) \geq x_t^L(\zeta)$, and the inequality is strict for a positive measure of $\zeta$. Consequently, we have proved that $\kappa_t^H > \kappa_t^L$, which immediately leads to $\Delta_t^\omega > 0$.

**Proof of Proposition 6** Next, we analyze the impact of credit intervention. Under Assumption 1, we can simplify the financing choices of H and L types as in (A24) and (A25). We first show that $\partial \Delta_t^K / \partial \bar{g} > 0$. According to (22), a sufficient condition is

$$\frac{\partial \kappa_t^j}{\partial \bar{g}} > 0.$$

According to equation (A24), $x_t^H(\zeta)$ monotonically increases in $\bar{g}$ (given $q_t^j$ and thus $\bar{\zeta}_t^H$). Furthermore, according to equation (A25), $x_t^L(\zeta)$ increases monotonically in $\bar{g}$ if either $\zeta \geq \underline{\zeta}_t^L$ or $\zeta < \underline{\zeta}_t^L$. By Assumption 1, L type firms have slackness in the continuation region $\zeta \geq \underline{\zeta}_t^j$ (i.e., $x_t^L(\zeta)$ is not affected by $\bar{g}$ for $\zeta \geq \underline{\zeta}_t^j$), so $\bar{g}$ does not affect the cutoff $\underline{\zeta}_t^L$ (see equation (17)). Taken together, we conclude that both $x_t^H(\zeta)$ and $x_t^L(\zeta)$ monotonically increase with $\bar{g}$, and therefore, $\kappa_t^H$ and $\kappa_t^L$, the integration of these two functions, also increase in $\bar{g}$. This leads to $\partial \Delta_t^K / \partial \bar{g} > 0$.

Next, we note that $\partial \Delta_t^\omega / \partial \bar{g} < 0$ is equivalent to

$$\frac{\partial (\kappa_t^L / \kappa_t^H)}{\partial \bar{g}} > 0 \Leftrightarrow \frac{\partial \kappa_t^L}{\partial \bar{g}} - \frac{\kappa_t^L}{\kappa_t^H} \frac{\partial \kappa_t^H}{\partial \bar{g}} > 0. \tag{A26}$$

Since $\kappa_t^H > \kappa_t^L$, a sufficient condition for (A26) is

$$\frac{\partial \kappa_t^L}{\partial \bar{g}} > \frac{\partial \kappa_t^H}{\partial \bar{g}}.$$

Again, we look at $x_t^H(\zeta)$ and $x_t^L(\zeta)$ for each $\zeta$. Given $\bar{d} + \bar{g} > \bar{\zeta}_t^H$, we have $(\bar{\zeta}_t^H - \zeta)^+ \leq \bar{d} + \bar{g}$ for any $\zeta \geq 0$, so government intervention does not directly affect H-type firms, with $\partial x_t^H(\zeta) / \partial \bar{g} = 0$. On the other hand, since $\hat{d}^L(\zeta)$ increases with $\bar{g}$ (see Lemma 4), and $\underline{\zeta}_t^L$ is not affected by $\bar{g}$, we find that $x_t^L(\zeta)$ as in (A25) weakly increases with $\bar{g}$, and the increase is strict for all $\zeta < \underline{\zeta}_t^L$. As a result, we get

$$\frac{\partial \int F(x_t^H(\zeta) + \zeta) dH(\zeta)}{\partial \bar{g}} = 0,$$

$$\frac{\partial \int F(x_t^L(\zeta) + \zeta) dH(\zeta)}{\partial \bar{g}} > 0.$$

The above naturally lead to the sufficient condition in (A26), and thus the result $\partial \Delta_t^\omega / \partial \bar{g} < 0$.

**Proof of Proposition 7.** The percentage output drop in a crisis is

$$\frac{(A^H K_t^H + A^L K_t^L) - (A^H K_{t-}^H + A^L K_{t-}^L)}{A^H K_{t-}^H + A^L K_{t-}^L} = \frac{A^H \omega_{t-} \kappa_{t-}^H + A^L(1-\omega_{t-})\kappa_{t-}^L}{A^H \omega_{t-} + A^L(1-\omega_{t-})}$$

where we denote the fractional decline of type-$j$ capital as

$$\kappa_{t-}^j \equiv \int_\zeta (1 - F(x_{t-}^j(\zeta) + \zeta)) dH(\zeta)$$

We note that $\kappa_{t-}^j$ is not affected by $\omega_{t-}$ but affected by $\bar{g}$, so we express it as $\kappa_{t-}^j(\bar{g})$.

Denote the target GDP drop as $\Delta y$. For simplicity, we drop the time-$t$ subscripts since we consider a stationary Markov equilibrium (note that $\omega$ denotes pre-crisis capital quality $\omega_{t-}$). Then we have

$$\frac{A^H \omega \kappa^H(\bar{g}) + A^L(1-\omega)\kappa^L(\bar{g})}{A^H \omega + A^L(1-\omega)} = \Delta y \tag{A27}$$

We are interested in how $\bar{g}$ is affected by the firm quality $\omega$, i.e., we want to know the derivative of $\bar{g}(\omega)$, defined by the above equation. Taking derivative over $\omega$ on both sides of A27, we obtain

$$\left( \left( A^H \kappa^H(\bar{g}) - A^L \kappa^L(\bar{g}) \right) + \left( A^H \omega \frac{d\kappa^H}{d\bar{g}} + A^L(1-\omega)\frac{d\kappa^H}{d\bar{g}} \right) \frac{d\bar{g}}{d\omega} \right) (A^H \omega + A^L(1-\omega))$$
$$- (A^H - A^L) \left( A^H \omega \kappa^H(\bar{g}) + A^L(1-\omega)\kappa^L(\bar{g}) \right) = 0$$

The terms that do not involve derivatives can be simplified as

$$\begin{aligned}
&(A^H \kappa^H(\bar{g}) - A^L \kappa^L(\bar{g}))(A^H \omega + A^L(1-\omega)) - (A^H - A^L) \left( A^H \omega \kappa^H(\bar{g}) + A^L(1-\omega)\kappa^L(\bar{g}) \right) \\
=& A^H A^L \kappa^H(\bar{g})(1-\omega) - A^H A^L \omega \kappa^L(\bar{g}) - A^H A^L (1-\omega)\kappa^L(\bar{g}) + A^H A^L \omega \kappa^H(\bar{g}) \\
=& A^H A^L \left( \kappa^H(\bar{g}) - \kappa^L(\bar{g}) \right)
\end{aligned}$$

Therefore, we get

$$A^H A^L \left( \kappa^H(\bar{g}) - \kappa^L(\bar{g}) \right) + \left( A^H \omega \frac{d\kappa^H}{d\bar{g}} + A^L(1-\omega)\frac{d\kappa^H}{d\bar{g}} \right) (A^H \omega + A^L(1-\omega)) \frac{d\bar{g}}{d\omega} = 0$$

$$\frac{d\bar{g}}{d\omega} = -\frac{A^H A^L \left( \kappa^H(\bar{g}) - \kappa^L(\bar{g}) \right)}{\left( A^H \omega \frac{d\kappa^H}{d\bar{g}} + A^L(1-\omega)\frac{d\kappa^H}{d\bar{g}} \right) (A^H \omega + A^L(1-\omega))}$$

By assumption, there is net cleansing effect, i.e., the change of firm quality is positive,

$$\Delta\omega = \frac{\omega}{\omega + (1-\omega)\frac{1-\kappa^L}{1-\kappa^H}} - \omega > 0$$

which is equivalent to $\kappa^H < \kappa^L$. We already show that $d\kappa^H/d\bar{g} < 0$ and $d\kappa^L/d\bar{g} < 0$ in the proof of Proposition 6. As a result, we obtain

$$\frac{d\bar{g}}{d\omega} < 0.$$

# B Additional Model Properties

## B.1 Dynamic Optimal Intervention

In Section 3.3, we consider the dynamics in an environment where agents do not expect intervention. This approach purges out any backward propagation of policy impact (i.e., future interventions affect the effectiveness and necessary scale of intervention in the current crisis through agents' expectations) and thereby only shows the strength of forward propagation of policy impact (i.e., intervention in the current crisis affects the scale of future interventions) which is the focus of our paper. Next, we consider an environment where the government optimally chooses the scale of intervention in every crisis, and agents have rational expectations of the policy plan. How will the backward propagation of policy impact agents' expectations and interact with the forward propagation in equilibrium? Can the government avoid the slippery slope of intervention by dynamically adjusting $\bar{g}$? These questions are important because the reality, in terms of agents' expectation of intervention, likely lies in between the situation in Section 3.3 and fully rational expectation. Moreover, in practice, the policy goal may also lie between containing the output drop in crises (the least sophisticated) and dynamically maximizing welfare (the most sophisticated).

Dynamically adjusting the scale of intervention can potentially improve efficiency. In our model, firm quality distribution, represented by $\omega_t$, evolves over time. When $\omega_t$ is low and there are many type-$L$ firms, the government would reduce $\bar{g}$, because the cost of type-$L$ firms' over-spending overweighs the benefit of relaxing the under-spending firms' financial constraints; in contrast, when $\omega_t$ is high, the government would increase $\bar{g}$. Below, we solve the optimal $\bar{g}_t = g^{dynamic}(\omega_t)$ through the dynamic optimization of welfare. Intuitively, the optimal policy is dependent on only the quality state variable, $\omega_t$, because the economy is scalable in the quantity state variable, $K_t$, and the intervention scale is about funding support per unit of capital. We show that even though dynamically adjusting the scale of intervention improves welfare, it cannot eliminate the persistent distortions on firm quality distribution and the slippery slope of intervention.

When the scale of government funding support depends on $\omega_t$, both $q_t^H$ and $q_t^L$ become time-varying and dependent on $\omega_t$. For $j \in \{H,\, L\}$, the capital value has the following law of motion:

$$\frac{dq_t^j}{q_{t-}^j} = \mu_{q,t-}^j dt + \Delta_{q,t-}^j dN_t.$$

With the capital value as a function of $\omega_t$, (i.e., $q_t^j = q^j(\omega_t)$), we obtain the drift and jump size:

$$\mu_{q,t-}^j = \frac{dq^j(\omega_{t-})}{d\omega_{t-}} \mu_t^\omega(\omega_{t-})dt,$$

and

$$\Delta_{q,t-}^j = \frac{q^j(\omega_{t-} + \Delta_t^\omega(\omega_{t-})) - q^j(\omega_{t-})}{q^j(\omega_{t-})},$$

The capital valuation equation is the same as (21), although now $E[dq_t^j] \neq 0$, unlike the economy with static credit intervention. Solving the model requires solving the following HJB equation for the $K_t$-scaled welfare function, $W(\omega_t)$:

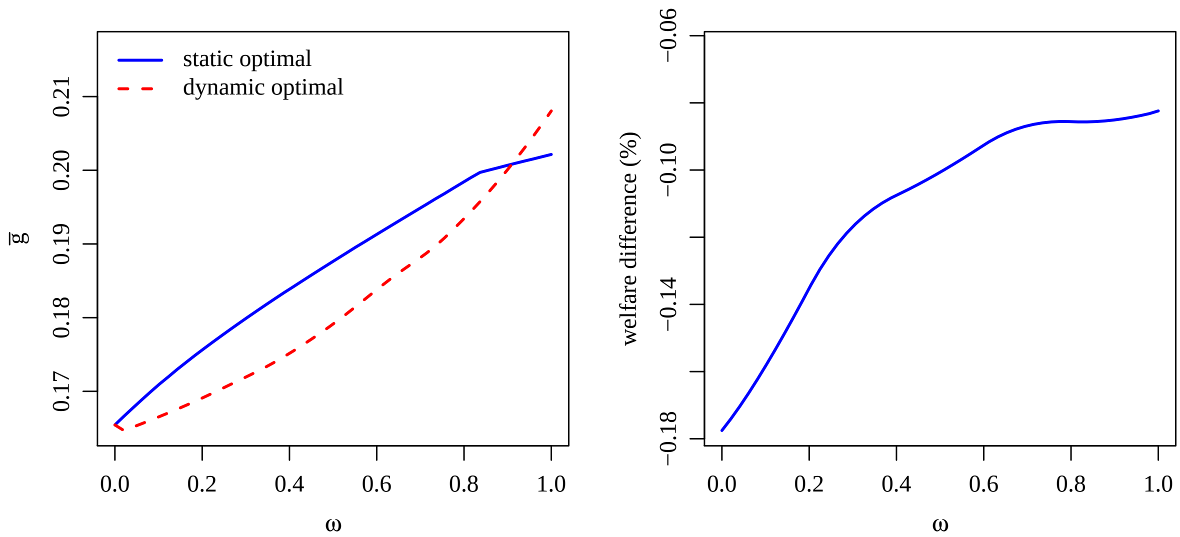
$$\begin{aligned} rW(\omega_{t-}) =& \omega_{t-}A^H + (1-\omega_{t-})A^L - \left(\omega_{t-}\bar{\iota}^H(\omega_{t-}) + (1-\omega_{t-})\bar{\iota}^L(\omega_{t-})\right) \\ &+ W(\omega_{t-})\mu^K(\omega_{t-}) + W'(\omega_{t-})\mu^\omega(\omega_{t-}) \\ &+ \lambda \max_{\bar{g}} \left[W(\omega_{t-} + \Delta^\omega(\omega_{t-}, \bar{g}))\left(1 + \Delta^K(\omega_{t-}, \bar{g})\right) - W(\omega_{t-}) - I(\omega_{t-}, \bar{g})\right] . \end{aligned} \tag{A28}$$

In comparison with the welfare HJB equation (27) under a constant $\bar{g}$, the last term on the right side of (A28) reflects the optimization over $\bar{g}$ given the firm quality distribution, represented by $\omega_{t-}$, that the economy carries into a crisis.

The first-order condition for the optimal $\bar{g}$ reveals the trade-off that the government faces:

$$\begin{aligned} &W'(\omega_{t-} + \Delta^\omega(\omega_{t-}, \bar{g}))\frac{\partial \Delta^\omega(\omega_{t-}, \bar{g})}{\partial \bar{g}}\left(1 + \Delta^K(\omega_{t-}, \bar{g})\right) + \\ &W(\omega_{t-} + \Delta^\omega(\omega_{t-}, \bar{g}))\frac{\partial \Delta^K(\omega_{t-}, \bar{g})}{\partial \bar{g}} - \frac{\partial I(\omega_{t-}, \gamma)}{\partial \bar{g}} = 0 \,. \end{aligned} \tag{A29}$$

The first term shows the negative impact of dampening the cleansing effect, $\frac{\partial \Delta^\omega(\omega_{t-}, \bar{g})}{\partial \bar{g}} < 0$, and its long-run effects are encoded in the marginal change of the present value of future consumption (i.e., the forward-looking welfare measure) per unit of capital over change in capital quality, $W'(\omega_{t-} + \Delta^\omega(\omega_{t-}, \bar{g}))$. The second term shows the positive impact of reducing $\bar{g}$ through the preservation of capital, $\frac{\partial \Delta^K(\omega_{t-}, \bar{g})}{\partial \bar{g}} > 0$. Each unit of capital saved by the government funding raises welfare by $W(\omega_{t-} + \Delta^\omega(\omega_{t-}, \bar{g}))$. The last term reflects the reduction of consumption due to the provision of government funding.




(a) Optimal government intervention (b) Difference in optimal welfare

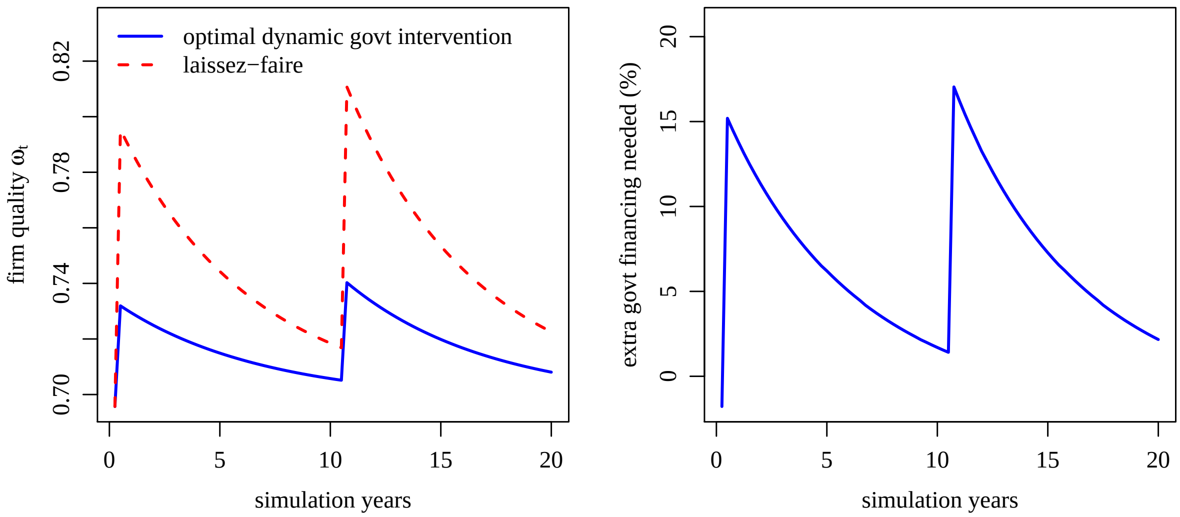
**Figure A1: Optimal dynamic intervention and welfare.** In panel (a), we plot the optimal dynamic government intervention $\bar{g}^*(\omega_t)$ as a function of the state $\omega_t$ at time $t$, and the optimal static government intervention $\bar{g}^*(\omega_0)$ as a function of initial state $\omega_0$. In panel (b), we show the percentage difference in welfare under the dynamic optimal policy versus the static optimal policy. A negative value indicates that welfare under dynamic optimal policy is lower.

To simplify the notation, we omit the time subscripts below. Equation (A29) implicitly defines the optimal $\bar{g}$ as a function of $\omega$. Once we solve the functions $q^H(\omega)$, $q^L(\omega)$, $W(\omega)$, and $\bar{g}(\omega)$, we obtain the time-$t$ values of the other endogenous variables as functions of $\omega$.

In equilibrium, the capital value, $q_t^j$ ($j \in \{H, L\}$), the welfare per unit of capital, $W_t$, and the optimal government intervention scale $\bar{g}_t$, jointly satisfy the equations (21), (A28), and (A29), and firms' financing decisions are given by equation (19).

Panel A of Figure A1 compares the dynamically adjusted $\bar{g}$ and the optimal constant $\bar{g}$ set at $t = 0$. Overall, they are close numerically. For large $\omega$, the dynamic $\bar{g}$ is larger because, given the downward trajectory of firm quality for large $\omega$, the government can reduce its funding supply when the fraction of lower-quality firms is higher. For small $\omega$, the dynamic $\bar{g}$ is smaller because, given the upward trajectory of firm quality for small $\omega$, the government can increase its funding supply when the fraction of higher-quality firms is higher.

Panel B of Figure A1 illustrates the welfare difference between the scenario of committed static policy versus dynamic optimal policy. Although the dynamic policy provides extra flexibility for policy adjustment, the lack of commitment can reduce welfare through potential dynamic




(a) Firm quality dynamics

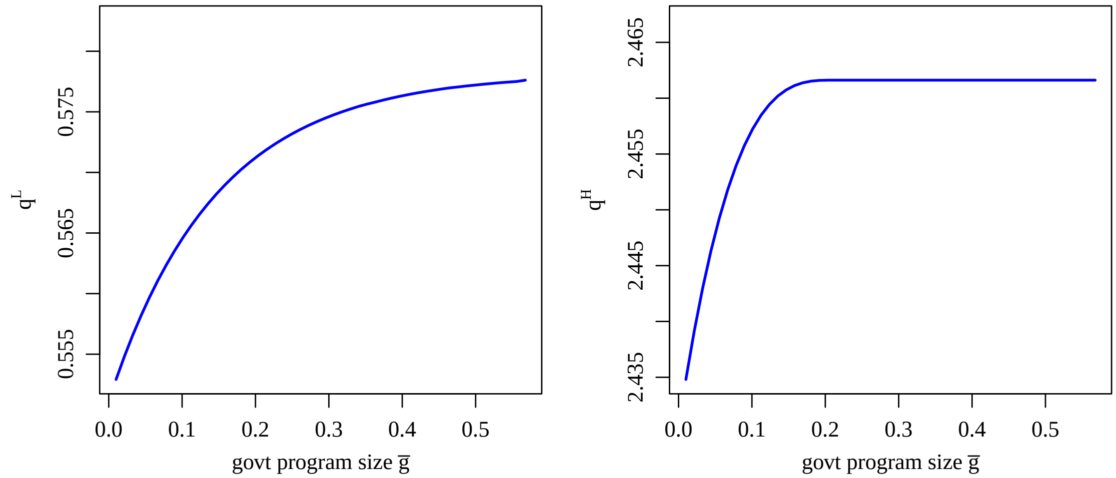
(b) Extra intervention required

**Figure A2: Slippery slope of credit intervention: Optimal dynamic policy versus laissez-faire benchmark.** In this figure, we compare two economies: In the laissez-faire economy, there is no intervention, and agents believe so; the govt-intervention economy is using the optimal dynamic intervention strategy $\bar{g}_t = g^{dynamic}(\omega_t)$ and agents rationally expect so. The starting state of the simulation is the average $\omega$ in the no-intervention economy. Then, we introduce two crises on the simulation path, one at year zero and the other at year ten. Panel (a) plots the simulated paths of $\omega_t$ for the two economies respectively. Next, at each point of time, we calculate the amount of intervention needed in the laissez-faire economy that is required to achieve the same percentage of GDP drop as the economy with optimal dynamic intervention if a crisis happens at that time. Then, we show the difference in intervention scale between the optimal dynamic intervention economy and the laissez-faire economy in panel (b).

inconsistency, and the net effect from our numerical evaluation is that the lack of commitment dominates.

Finally, in Figure A2, we illustrate the comparison between an economy with optimal dynamic intervention versus an economy without government intervention. The goal is to analyze how much extra intervention is needed in the dynamic-intervention economy compared to the no-intervention economy to achieve the same goal of containing GDP drop to within 10% in crises. Different from Figure 7, in the dynamic-intervention economy, agents correctly expect interventions during future crises. However, since crisis realizations are still surprises ($dN_t$ shocks are not predictable), each crisis still widens the gap between the intervention economy and the laissez-faire economy, causing extra intervention needed for the intervened economy.[40] Therefore, the optimal dynamic

[40] Also note that in Figure A2(b) the extra intervention at the beginning is negative, because the initial state $\omega_0$ is the same across two economics, while capital values are higher in the intervention-economy, so less intervention is needed




(a) $q^L$ (b) $q^H$

**Figure A3: Credit intervention and capital values.** This figure illustrates the impact of a static government intervention $\bar{g}$ on capital value $q^H$ and $q^L$.

adjustment of government policy does not eliminate the slippery slope of intervention.

## B.2 Capital Value and the Stationary Distribution

In this subsection, we illustrate the effect of government intervention through the impact on capital values and the stationary distribution of $\omega_t$. We note that the stationary distribution is affected not only by crisis-time jumps but also by normal-time investment that is directly driven by capital values. Figure A3 (a) and (b) illustrate the impact of government intervention on capital values. Since government intervention improves crisis-time value $\pi^j$ (see equation (21)), it increases the capital value $q^j$. However, the impact on $q^H$ is limited when $\bar{g}$ expands beyond a certain threshold, because the borrowing demand from type-$H$ firms is satiated once they reach the efficient level of spending. On the other hand, $q^L$ further increases, because type-$L$ firms may over-borrow to survive in future crises and the extra subsidy from a larger $\bar{g}$ is priced in the capital valuation.

In Figure A4, we illustrate the stationary distribution of $\omega_t$ in two economies, one without intervention ($\bar{g} = 0$), and the other with baseline intervention ($\bar{g} = 0.14$). We find that the distribution of $\omega_t$ significantly shifts to the leftwards in the latter, reflecting a long-run reduction of firm quality.

in the intervention-economy. Later, the difference in $\omega_t$ becomes dominant and overcomes this capital valuation effect.

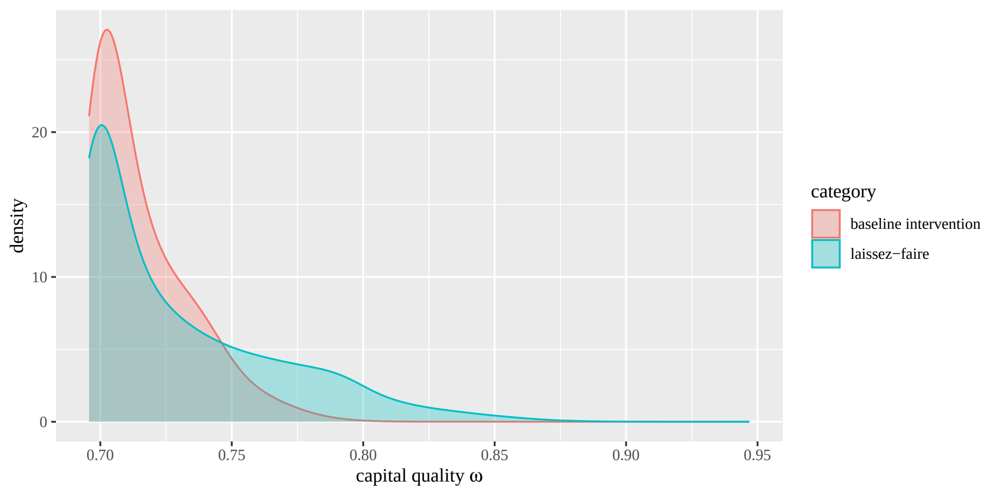



**Figure A4: Stationary distribution of** $\omega_t$. This figure illustrates the stationary distribution of $\omega_t$, under two different scenarios of government intervention: laissez-faire with $\bar{g} = 0$, and baseline intervention with $\bar{g} = 0.14$.

## B.3 Intervention without Market-Based Pricing

In this section, we discuss the lack of differentiation in credit pricing and show that government intervention dampens the cleansing effect of crises and the slippery slope of credit intervention. Such lack of differentiation in credit pricing is a feature in many government programs, including the Main Street Lending Program and the Paycheck Protection Program. To clarify the channel, we demonstrate it in a simplified setting. Specifically, we shut down the funding-demand side friction (i.e., the incentive to over-borrow and strategically default) via $\beta = 0$. In our baseline model with market-based pricing discrimination, this would imply that credit intervention does not induce inefficiency. Moreover, We assume that both $\bar{d}$ and $\bar{g}$ are large enough so that the funding constraints do not bind (i.e., all firms can potentially spend on survival at the first-best levels).

We first consider the laissez-faire economy. Denote the borrowing from private-sector as $d^j(\zeta)$ and borrowing from government as $g^j(\zeta)$, with type $j \in \{L, H\}$. For a type-$j$ firm, given $\zeta$, the optimization problem is

$$\max_{d^j} F(d^j + \zeta)\left[q^j - (1 + r(\zeta, d^j, q^j))d^j\right],$$

with private-sector break-even condition

$$F(d^j + \zeta)(1 + r(\zeta, d^j, q^j))d^j = d^j$$

Therefore, the equivalent problem is

$$\max_{d^j} F(d^j + \zeta)q^j - d^j$$

The solution is

$$d^j(\zeta) = \bar{\zeta}^j - \zeta$$

with $\bar{\zeta}^j$ defined as

$$F'(\bar{\zeta}^j)q^j = 1$$

Because $q^H > q^L$, the above solution indicates that $\bar{\zeta}^H > \bar{\zeta}^L$ so that $d^H(\zeta) > d^L(\zeta)$. By definition,

$$\kappa^j = \int F(d^j(\zeta) + \zeta)dH(\zeta)$$

Consequently, we have $\kappa^H > \kappa^L$ and $\Delta^\omega > 0$. Crises feature cleansing effect.

Next, we consider intervention. Let the interest rate charged by the government be $\bar{r} > 0$, which is the same across all firms, and for simplicity, constant over time. This implies that the equilibrium capital value is constant, similar to the stationary equilibrium in the main text. The same rate for all firms is the actual practice in government programs including MSLP and PPP. Firms may utilize the funds to (over-)spend on their own survival probability or deposit at the market-based interest rates. Note that we allow firms to lend to one another. If $\bar{r}$ is too high for one firm, the firm may instead borrow from other firms. Forbidding such reallocation of funds results in great inefficiency induced by the lack of interest-rate discrimination in government credit support. Under the same rate set by the government, the optimization problem of the firm becomes

$$\max_{g^j \geq 0, d^j} F(d^j + g^j + \zeta)\left[q^j - (1 + r(\zeta, d^j, g^j, q^j))d^j - (1 + \bar{r})g^j\right], \tag{A30}$$

with private-sector break-even condition

$$F(d^j + g^j + \zeta)(1 + r(\zeta, d^j, g^j, q^j))d^j = d^j. \tag{A31}$$

With this expression, we can rewrite the optimization problem as

$$\max_{d^j \geq 0, g^j \geq 0} F(d^j + g^j + \zeta)\left(q^j - (1 + \bar{r})g^j\right) - d^j. \tag{A32}$$

Denote the objective function as $J(d^j, g^j; \zeta, q^j)$. Then, the derivative over $d^j$ is

$$J'_d(d^j, g^j; \zeta, q^j) = F'(d^j + g^j + \zeta)\left(q^j - (1+\bar{r})g^j\right) - 1 \tag{A33}$$

and the derivative over $g^j$ is

$$J'_g(d^j, g^j; \zeta, q^j) = F'(d^j + g^j + \zeta)\left(q^j - (1+\bar{r})g^j\right) - F(d^j + g^j + \zeta)(1+\bar{r}) \tag{A34}$$

We note that since $1 + \bar{r} > 1$, there exists $d^j$ such that $F(d^j + g^j + \zeta)(1 + \bar{r}) = 1$ and that we can simultaneously have $J'_d = 0$ ad $J'_g = 0$, i.e., an interior solution for the problem. The interior solution is $J'_g(d^j, g^j; \zeta, q^j) = J'_d(d^j, g^j; \zeta, q^j) = 0$, which implies

$$F(d^j + g^j + \zeta)(1+\bar{r}) = 1, \tag{A35}$$

$$d^j + g^j = F^{-1}(\frac{1}{1+\bar{r}}) - \zeta. \tag{A36}$$

Plugging (A36) into $J'_d = 0$, we obtain

$$g^j = \frac{1}{1+\bar{r}}\left(q - \frac{1}{F'(F^{-1}(\frac{1}{1+\bar{r}}))}\right)$$

which is not affected by $\zeta$. The private-sector borrowing is then

$$d^j = F^{-1}(\frac{1}{1+\bar{r}}) - \frac{1}{1+\bar{r}}\left(q - \frac{1}{F'(F^{-1}(\frac{1}{1+\bar{r}}))}\right) - \zeta.$$

Since

$$\kappa^j = \int F(d^j(\zeta) + g^j(\zeta) + \zeta)dH(\zeta)$$

According to (A36), $\kappa^H = \kappa^L$. Thus, the cleansing effect is completely eliminated, and we have the following proposition.

**Proposition 8 (Lack of differentiation in pricing and the dampening of cleansing effect)** *Assume $\beta = 0$ and financing capacity constraints are not binding. If the government charges the same in-*

*terest rate $\bar{r} > 0$ to all firms, then the cleansing effect of crisis is completely eliminated,*

$$\Delta_t^{\omega} = 0. \tag{A37}$$

*This dampening of cleansing effect is inefficient for two reasons: First, it lacks differentiation between H and L firms, over-subsidizing L-type firms but not enough for H-type firms; Second, it lacks differentiation among same-type firms with different credit risks (due to heterogeneous $\zeta$), causing over-borrowing for high-$\zeta$ firms but not enough borrowing for low-$\zeta$ firms.*

## B.4 Zombie Firms and the Amplification Effects

We lay out an alternative setup that incorporates zombie firms. By comparing this alternative setup with our baseline model in the main text, we illustrate how the presence of zombie firms amplifies the distortionary effects of policy intervention and makes such effect more persistent.

The operation of zombie firms relies on external resources. Zombie firms can be modeled as firms with a negative productivity. In our model, the productivity, $A^H$ or $A^L$, represents the value-added, i.e., the value created net off the resources consumed in the production process. In the following, we modify how type-$L$ firms are modeled to make them zombies (the modeling of type-$H$ firms follows the main model). A type-$L$ firm is a zombie firm with negative productivity, i.e., $A^L < 0$. The firm behaves differently from the non-zombie type-$L$ firms in the main text.

In normal times, type-$L$ firms consider a different value per unit of capital, $\widetilde{q} > 0$, which is a distorted Tobin's q. Under $A^L < 0$, the firm has a negative Tobin's q, i.e., $q_t^L < 0$ as it is the present value of net cash flows ($< 0$) that the firm generates. When making investments, the firm considers, $\widetilde{q}$ ($> 0$), which captures the fact that the firm owners derive value from operating the firm, such as salaries for the board members and managers and certain private benefit from managing the firm and its labor force. The distorted value $\widetilde{q}$ also captures the benefits that accrue to other stake holders, for example, bankers who benefit from evergreening loans (Acharya, Crosignani, Eisert, and Steffen, 2022). In our setup, we assume that $\widetilde{q}$ is smaller than $q^L$ in our main model, and thus smaller than $q^H$. At time $t$, a type-$L$ firm with $k_t$ units of capital chooses the investment rate, $\iota_t^L$:

$$\max_{\iota_t^L} \widetilde{q}\, k_t \iota_t^L - \Phi(\iota_t^L, k_t)\,, \tag{A38}$$

where, as in the main text, we adopt the quadratic investment cost function from Hayashi (1982):

$$\Phi(\iota_t^L, k_t) = \left(\iota_t^L + \frac{\theta}{2}\iota_t^{L\,2}\right) k_t. \tag{A39}$$

Under this functional form, we obtain the following choice of investment:

$$\iota_t^j = \frac{\widetilde{q} - 1}{\theta}\,. \tag{A40}$$

Relative to our main setup, here $q_t^L$ is replaced by $\widetilde{q}$.

In crises, our analysis in the main text carries through with $\widetilde{q}$ replacing $q_t^L$ in the firm's objective function (12), the definition of $\overline{\zeta}_t^L$ in (14), and the definition of strategic default threshold $\underline{\zeta}_t^L$ in (17). We note that $\overline{\zeta}_t^H > \overline{\zeta}_t^L$ in Lemma 1 carries through as $q^H > \widetilde{q}$. Moreover, $\underline{\zeta}_t^H < \underline{\zeta}_t^L$ in Lemma 2 still holds under $q^H > \widetilde{q}$, and Lemma 3 holds where obtain $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$ under $q^H > \widetilde{q}$. Therefore, Proposition 1 can be rewritten with $\widetilde{q}$ replacing $q_t^L$, and all the properties carry through. As a result, our results in Section 2.3 and 2.4 are still valid. Therefore, modifying the way we model type-$L$ firms to make them zombie firms does not qualitatively change the properties of our model.

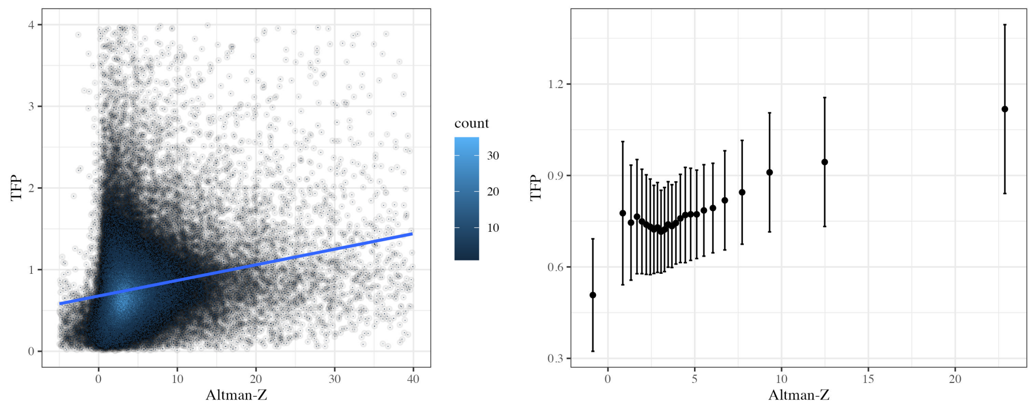
By making type-$L$ firms into zombie firms, this alternative setup can amplify the distortionary effect of policy intervention. From a welfare perspective, the damage from dampening the cleansing effects of crises is now greater, as the productivity gap between type-$H$ and type-$L$ firms (and consequently, the capital valuation gap) is larger. In normal times, the continuing operation and investments of zombie firms are directly costly. In crises, policy intervention induces a greater degree of inefficiency because, under $\widetilde{q} < q^L$, more type-$L$ firms strategically default than in the main model (capital value is inversely related to strategic default incentive as shown in Lemma 2), which in turn suggests more firms will over-spend on improving survival probability. As pointed out in Section 2.3, the source of inefficiency is the over-spending by firms that strategically default. By enabling this force, policy intervention induces more inefficient spending in crises.

In crises, government liquidity support reduces $\omega_t$ (fraction of firms being type-$H$) even more relative to the laissez-faire benchmark. Under $\widetilde{q} < q^L$, more type-$L$ firms strategically default, and these firms simply maximize borrowing from both private-sector creditors and the government (they always exhaust the borrowing limit set by the government). Therefore, given any borrowing limit $\bar{g}$, this alternative model features a greater take-up rate, which then translates into a greater fraction of type-$L$ firms saved by government liquidity support. The slippery slope still emerges and becomes more potent as the distortionary effects of policy intervention are now more greater. As shown in Proposition 7, the necessary scale of intervention depends on $\omega_{t-}$ (fraction of firms being type-$H$) right before the arrival of the next crisis, and the slippery slope arises because intervention in the current crisis reduces $\omega_{t-}$ for the next crisis. Given that policy intervention now dampens the cleansing effect of crises (i.e., reducing $\omega_t$ that the economy carries out of the current crisis) more strongly under $\widetilde{q} < q^L$, the slippery slope of intervention is more potent.

This alternative setup does not feature zombie firms crowding out other firms in product or factor markets. Such crowding-out is important and emphasized in the literature on zombie firms and zombie lending (e.g., Acharya, Lenzu, and Wang, 2021). We do not incorporate this channel in the alternative setup to stay as close as possible to our model in the main text for easy comparison.

# C Additional Empirical Evidence

## C.1 Firm Quality and Credit Quality




(a) Scatter Plot (b) Binscatter Plot

**Figure A5: Credit quality v.s. firm productivity.** We use the standard Altman-Z score to measure credit quality, and construct the cross-section of TFP following İmrohoroğlu and Tüzel (2014). In panel (a), we restrict the plot range to $0 \leq \text{TFP} \leq 4$ and $-5 \leq \text{Altman-Z} \leq 40$, which removes extreme outliers ($< 0.02\%$ of the sample). In panel (b), we show 25 bins of Altman-Z and plot the mean of TFP with two standard deviation bars on each side.

In our model, within each type $j \in \{H, L\}$, there is variation in the survival probability that is driven $\zeta$ and affects firms' credit worthiness (i.e., default probability). Although firm credit quality is observable via credit ratings, productivity is much more challenging to measure especially for the government that may lack the expertise in analyzing firm productivity. In a world where these two dimensions perfectly align, the government might rely on credit quality to know about firms' productivity and differentiate firms accordingly in its credit programs, mitigating the distortionary effects. Empirically, this is not the case: the correlation between credit quality and productivity is positive but far from perfect, consistent with our two-dimensional heterogeneity in type and $\zeta$.

We measure the cross section of firm TFP using the codes provided by İmrohoroğlu and Tüzel (2014) that apply the method originally developed in Olley and Pakes (1992). The methodology uses investment data to deal with classic simultaneity bias where the firm's factor input decision is influenced by TFP, and it also deals with firm exit through the correction based on selection probability estimation. We measure firm credit quality using Altman Z score (Altman, 1968), which is a combination of different accounting ratios to predict corporate bankruptcy. A higher Z score indicates better credit quality. Our data sample runs at yearly frequency from 1963 to 2020.

We plot the cross-section of Z score against TFP in Figure A5. As shown by the scatter plot in panel (a), for certain levels of TFP, there is a wide distribution of credit quality. In panel (b), we report a binscatter plot. The wide error bars suggest that the correlation is far from perfect.

## C.2 Firm Quality and Financing Capacity

In our model, the government provides equal liquidity support to both high- and low-productivity firms. As a result, such intervention improves the survival probability of low productivity (type-$L$) firms more than that of high productivity (type-$H$) firms, dampening the cleansing effect of crises. This is because type-$L$ firms can raise less credit from private-sector creditors than type-$H$ firms so the marginal impact of government support is greater for type-$L$ firms. Next, we provide evidence that low-productivity firms have smaller external financing capacity than high-productivity firms.

In the corporate finance literature, two indices that measure the tightness of firms' financial constraint have been widely adopted: Kaplan and Zingales (1997) (KZ) and Whited and Wu (2006) (WW). They measure the gap between targeted level of investment (funding needs) and financing capacity (funding availability). Since the 1980s, extensive empirical studies have been devoted to measure the tightness of financial constraints as it indicates the degree of inefficiency in funding allocation in the economy. KZ and WW indices differ in that the KZ index requires a measure of $Q$, i.e., proxy for investment opportunities (Hayashi, 1982), while the WW index does not.

The KZ index specifies a linear combination of firm characteristics as a proxy for a firm's probability of being financially constrained. Firm characteristics include cash flow/capital, total debt/total assets, dividend payout/capital, cash holding/capital, and average $Q$ (market value/book value). Capital is measured as PP&E, which refers to property, plant, and equipment (i.e., a firm's physical capital). Intuitively, a firm is more financially constrained if it has lower cash flows, more debt in place, less dividend payouts, and less cash on hand. Moreover, if a firm has higher average $Q$, its investment needs are strong and thus it is more likely to be financially constrained. We follow Lamont, Polk, and Saaá-Requejo (2001) to construct the KZ index.[41] The Whited-Wu (WW) index measures the Lagrange multiplier of financial constraint through a linear combination of firm and industry characteristics (see Section 1.4 of Whited and Wu (2006)), including cash flow/total assets, long-term debt/total assets, dividend/total assets, log(total assets), industry sales growth, and firm sales growth. We follow Whited and Wu (2006) to construct the WW index.

[41]The original KZ index was applied to a subset of firms. Lamont, Polk, and Saaá-Requejo (2001) broaden the application to all firms in the CRSP-Compustat database of publicly listed corporations.

An index of financial constraint tightness depends on both funding needs and funding availability. Given the same financing capacity, firms that want to invest more are more financially constrained, while given the same investment needs, firms with smaller financing capacity are more constrained. For our purpose, we need to isolate the financing capacity component. We regress the financial constraint index of a firm on its $Q$ that drives its investment needs. The residual captures the firm's financing capacity. For robustness, we consider three versions of $Q$: "average $Q$" given by the ratio of a firm's market value to book value (commonly referred to as Tobin's q in the empirical literature), "total $Q$" that accounts for intangible capital (Peters and Taylor, 2017), and "marginal $Q$" from Gala, Gomes, and Liu (2022) that is robust to mis-specification of factors driving firms' evaluation of their investment opportunities such as their investors' stochastic discount factor and their production and investment technologies.

Specifically, we run firm-year panel regressions according to the following specification:

$$\text{financial constraint index}_{i,t} = \alpha_i + \delta_t + \beta * Q_{i,t} + \epsilon_{i,t},$$

where $\alpha_i$ is a firm fixed effect, $\delta_t$ is a time fixed effect, $Q_{i,t}$ is a measure of $Q$. Since the $Q$ measure controls for investment needs, we use the residual as proxy for funding availability:

$$\text{financial capacity index}_{i,t} \equiv -\epsilon_{i,t}.$$

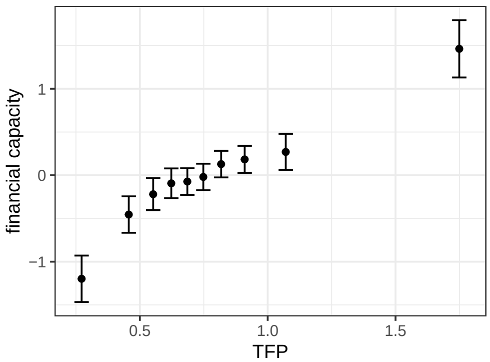
Note that we take add a minus sign to the residual because an increase in financing capacity reduces the tightness of financial constraint. Our sample is the standard Compustat/CRSP database of publicly listed companies at annual frequency, and it starts in 1963 and ends in 2020 in line with our TFP sample period. We restrict our sample to firms with positive total asset values, positive PP&E, and positive sales. To avoid extreme values, we winsorize all $Q$-measures and also the TFP measure by $1\%$ and $99\%$ quantile. The sample size differs across the three measures of $Q$ because different measures require different inputs. We compute the total $Q$ following Peters and Taylor (2017) and obtain the marginal q from the authors of Gala, Gomes, and Liu (2022).

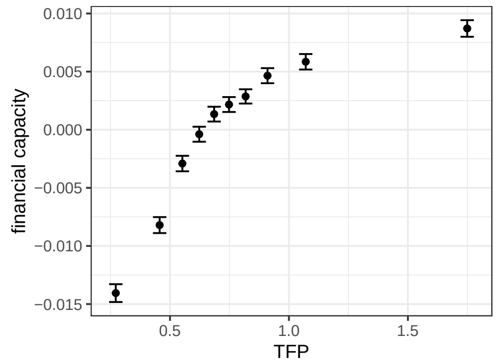
Since we have two measures of financial constraint and three measures of Q, we have $2\times3$=6 measures of financing capacity. In Figure A6, we illustrate the relation between firm TFP, which is constructed in the previous subsection, and the financing capacity measures. Our model suggests that higher productivity is associated with greater financing capacity. Indeed, across all six different measures of financing capacity, there is a strong positive correlation between TFP and financial

**Table A1: TFP and financing capacity: Regressions analysis**. Financing capacity is measured as the minus of various financial constraint indexes residualized by a measure of $Q$. "KZ index" is the Kaplan-Zingales Index in Kaplan and Zingales (1997). "WW index" is the Whited-Wu index in Whited and Wu (2006). To residualize these financial constraint indices and obtain measures of financing capacity, we consider three versions of $Q$: "average $Q$" given by the ratio of market value of book value, "total $Q$" that accounts for intangible capital (Peters and Taylor, 2017), and "marginal $Q$" that is robust to mis-specification of firm owners' stochastic discount factor and firms' technologies (Gala et al., 2022). We regress financing capacity on TFP controlling for firm and time fixed effects. Standard errors are clustered by firm and year and are reported in parentheses, with significance level indicated by * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$. Our data sample is at annual frequency from 1963 to 2020. The marginal $Q$ measure in Gala et al. (2022) has missing values and therefore a smaller sample size.

| | Dependent variable: financial capacity index | | | | | |
|---|---|---|---|---|---|---|
| | −residualized KZ index using | | | −residualized WW index using | | |
| | average Q | marginal Q | total Q | average Q | marginal Q | total Q |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| TFP | 4.786*** | 3.562*** | 4.756*** | 0.037*** | 0.041*** | 0.035*** |
| | (0.618) | (0.442) | (0.618) | (0.002) | (0.002) | (0.002) |
| Firm Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes |
| Time Fixed Effect | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 133,253 | 31,083 | 132,863 | 133,253 | 31,083 | 132,863 |
| $R^2$ | 0.010 | 0.012 | 0.010 | 0.054 | 0.068 | 0.048 |

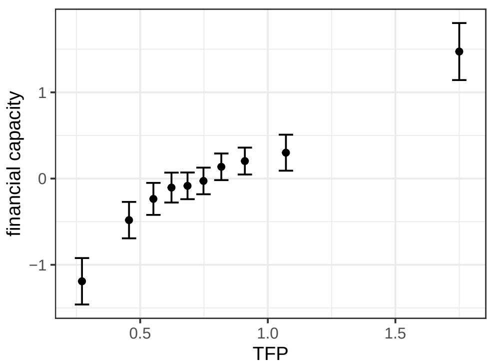
capacity. In Table A1, we regress the financing capacity index on TFP controlling for firm and time fixed effects. We find that the coefficient of TFP is positive and highly significant. Overall, these results support the key feature of our model that low-quality firms have smaller financing capacity than high-quality firms and government intervention narrows this gap.

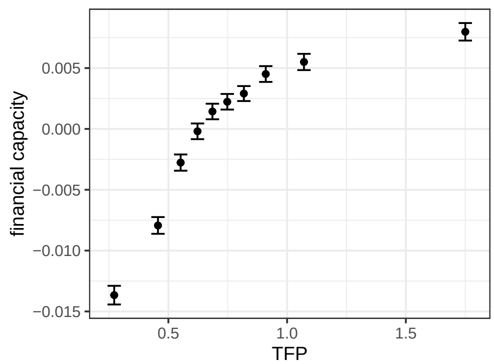



(a) $-$KZ index residualized by average $Q$

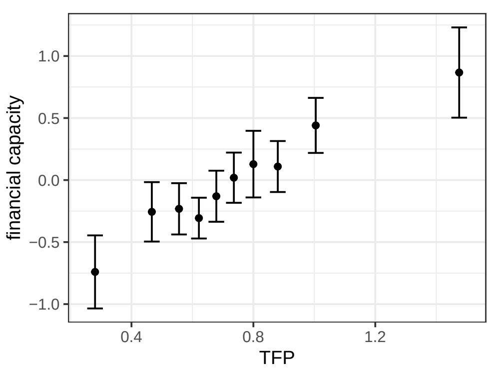
(b) $-$WW index residualized by average $Q$

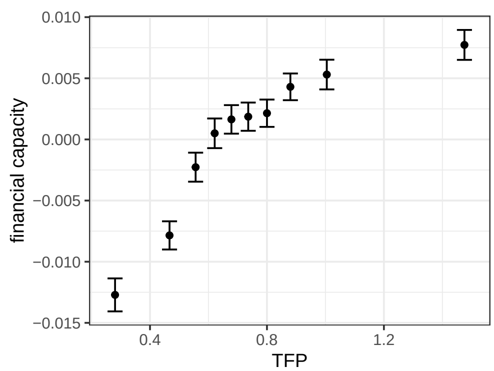



(c) $-$KZ index residualized by total $Q$

(d) $-$WW index residualized by total $Q$



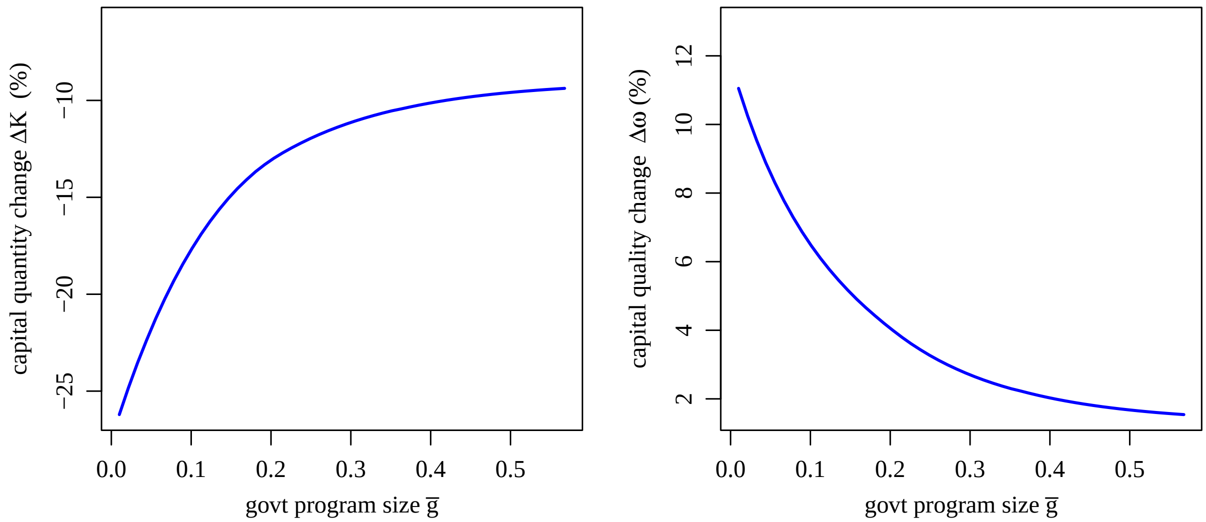

(e) $-$KZ index residualized by marginal $Q$

(f) $-$WW index residualized by marginal $Q$

**Figure A6: TFP and financing capacity**. Financing capacity is measured as the minus of various financial constraint indexes residualized by a measure of $Q$. "KZ index" is the Kaplan-Zingales Index in Kaplan and Zingales (1997). "WW index" is the Whited-Wu index in Whited and Wu (2006). To residualize these financial constraint indices and obtain measures of financing capacity, we consider three versions $Q$: "average $Q$" given by the ratio of market value of book value, "total $Q$" that accounts for intangible capital (Peters and Taylor, 2017), and "marginal $Q$" that is robust to mis-specification of firm owners' stochastic discount factor and firms' technologies (Gala et al., 2022). Firm-year observations are classified by the firm's TFP into ten bins, and for each bin, we show the average value of financing capacity and the band of two standard deviations. Our data sample is at annual frequency from 1963 to 2020.

# D Additional Quantitative Results

## D.1 Impact of Firm Entry



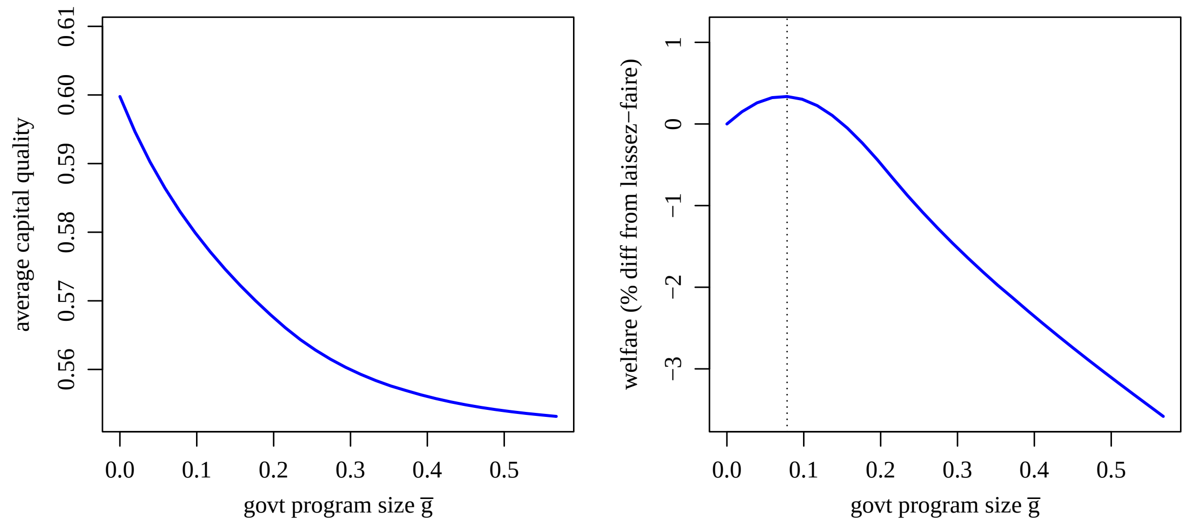

(a) Credit intervention and capital quantity

(b) Credit intervention and capital quality

**Figure A7: Credit intervention impact: Capital quantity vs. quality (under a larger $\eta$).** This figure illustrates how $\bar{g}$ affects $\Delta K_t$ and $\Delta\omega_t$ in a crisis, under larger $\eta$. The calculation requires the pre-crisis $\omega_{t-}$ which we set to the average value $\bar{\omega}$. This figure contrasts with Figure 2 in the main text.

The parameter $\eta$ governs the rate of firm entry is calibrated to generate entry rate in data. The reason to have firm entry is to maintain the stationarity of $\omega_t$, the fraction of firms being type-$H$ (high-quality). Absent from firm entry, $\omega_t$ drifts towards $1$, because type-$H$ firms grow capital at a higher rate than type-$L$ firms under $q^H > q^L$. Adding exogenous entry pulls $\omega_t$ away from $1$. Therefore, when we increase $\eta$, the whole process of $\omega_t$ shifts downward, further away from $1$. In the following, we conduct sensitivity analysis for $\eta$ and show that the qualitative dynamics under a 50% higher $\eta$ remain the same and the quantitative impact of government intervention is only slightly larger. The mechanisms can be easily understood from the fact that increasing $\eta_t$ causes the process of $\omega_t$ to shift downward. First, note that $q^H$ and $q^L$ are not affected. The entry of new firms does not interact with the valuation of existing capital, as shown in equation (21).[42]

[42] Our model does not feature the crowding out of productive firms by unproductive firms in product or factor markets (e.g., Caballero and Hammour, 1994).



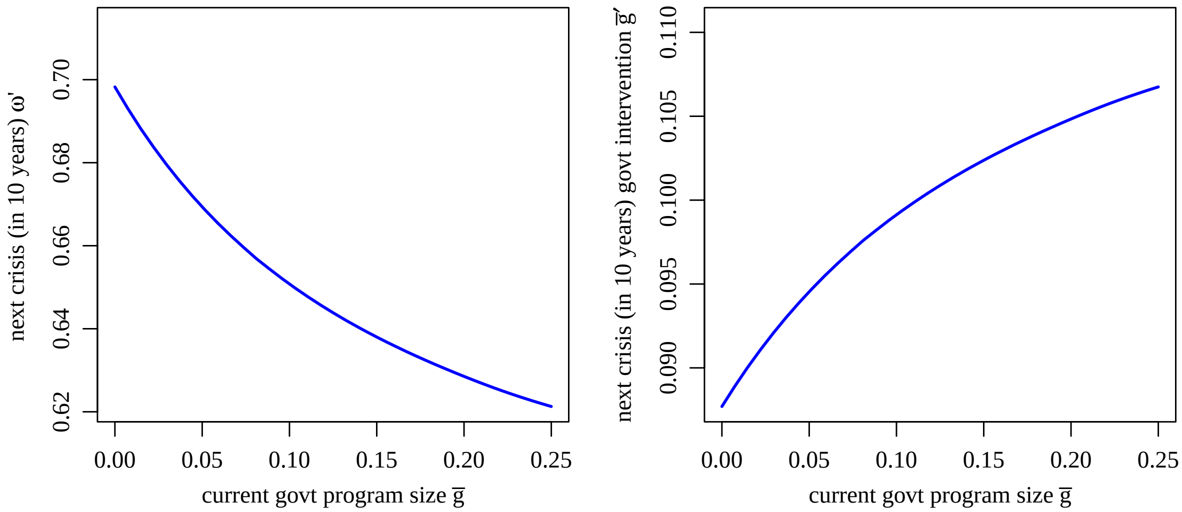

(a) Average capital quality

(b) Welfare improvement

**Figure A8: Credit intervention and welfare (under a larger $\eta$).** In panel A, we plot the average firm quality $\bar{\omega}$ as a function of government intervention $\bar{g}$. For each $\bar{g}$, we solve the model again and calculate the average of simulated $\omega_t$ as $\bar{\omega}(\bar{g})$. In panel B, we show the welfare difference $W(\bar{\omega}(\bar{g});\bar{g})/W(\bar{\omega}(0);0) - 1$ as a function of government intervention $\bar{g}$. This figure contrasts with Figure 4 in the main text.

In Figure A7, we plot the impact of intervention scale, $\bar{g}$, on capital quantity ($K_t$) and quality ($\omega_t$). Comparing Figure A7 with Figure 2 in the main text, we find that a larger $\eta$ increases the fraction of firms being type-$L$ and therefore increases rate of capital destruction in crises mechanically (see panel A) as type-$L$ firms on average have smaller financing capacity and can spend less on surviving the crisis than type-$H$ firms (see Section 2). This force—type-$L$ firms can obtain less financing than type-$H$ firms—also mechanically leads to a stronger cleansing effect of crises under a higher $\eta$ in a laissez-faire economy, as shown in panel B when $\bar{g}$ is set to zero. Our interest is on policy intervention distorts firm quality. In panel B, as we increase $\bar{g}$, intervention dampens the cleansing effect just as under the baseline value of $\eta$ in panel B of Figure 2. The quantitative results differ only slightly, suggesting that how intervention affects $\omega_t$ is crises is not sensitive to $\eta$.

In Figure A8, we show how government intervention affects firm quality over the long run (long-run average $\omega_t$) and welfare in this economy with a higher $\eta$. In panel A, we plot the average $\omega_t$ under different levels of intervention. The average is calculated with the stationary distribution of $\omega_t$ that depends on the normal-time drift (firms' investments), firm entry, and Poisson shocks (crises). With more type-$L$ firms under a higher $\eta$, $\omega_t$ is now mechanically lower than in the




(a) Firm quality entering the next crisis

(b) Required intervention in the next crisis

**Figure A9: Intervention pass-through across crises (under a larger $\eta$).** We show how $\bar{g}$ in the current crisis affects capital quality entering the next crisis, $\omega'$, and intervention needed, $\bar{g}'$, in the next crisis to contain output drop at $\underline{y}$. This limit $\underline{y}$ is set so that the starting point of the curve in Panel B, denoted by $\bar{g}'(\bar{g} = 0)$, is the same as Figure 6 in the main text. The next crisis happens ten years after the current one. Agents expect no intervention (pass-through is only due to forward propagation). The current crisis happens at $\omega$ equal to the average value of $\omega_t$ in the laissez-faire economy.

baseline calibration. However, in comparison with panel A of Figure 4 in the main text, the impact of intervention on the long-run average $\omega_t$ is only slightly bigger under a higher $\eta$, as represented by the degree of descent on the y-axis as we increases $\bar{g}$ from the left to the right on the x-axis. In panel B shows the impact on welfare. The range of welfare-improving $\bar{g}$ is smaller now under a higher $\eta$ relative to panel B of Figure A8. This is an intuitive result. The inefficiency from intervention comes from over-borrowing and over-spending, and such behavior is more prominent among type-$L$ firms as in the main model. A higher $\eta$ pushes down $\omega_t$ and amplifies such inefficiency.

In Figure A9, we show how intervention affects future interventions. To be comparable with Figure 6 in the main text, we set the policy goal to contain output drop at $\underline{y}$ where $\underline{y}$ is set so that the starting point the curve in panel B, i.e., the size of next intervention when the current invention scale is zero, denoted by $\bar{g}'(\bar{g} = 0)$, is the same as in panel B of Figure 6 in the main model. We find that an increase of current intervention $\bar{g}$ from 0 to 0.14 (which maps in our model to the scale of intervention during the Covid-19 pandemic) leads to an increase of intervention scale with a pass-through rate of about 12%, i.e., each one dollar of intervention per unit of capital in the current crisis generates 12 cents extra intervention per unit of capital in the next crisis should it happen ten years later. Comparing with the baseline results in Figure 6, we find that this doubles the effect of the slippery slope. The main intuition is again that increasing $\eta$ pushes down $\omega_t$, so with more type-$L$ firms in the economy, policy intervention has stronger distortionary effects.

## D.2 Introducing Firm-Type Transitions

In our main model, a firm's type does not change over time. In this extension, we allow firm type to switch from $H$ to $L$ or $L$ to $H$ at idiosyncratic Poisson times with intensity $\tilde{\eta}$.

The impact is that the wedge between $q^H$ and $q^L$ narrows as, intuitively, when a type-$H$ firm can become type-$L$ and vice versa, the capital values of two types are pulled together. As shown in Section 2, both the cleansing effects of crises and the distortionary effects of policy intervention hinge on the wedge between $q^H$ and $q^L$. The effect of introducing type transition is quantitatively the same as directly reducing the productivity wedge, $A^H - A^L$, between type-$L$ and -$H$ of the firms. Therefore, introducing type transition at Poisson rate $\tilde{\eta}$ to narrow the wedge between $q^H$ and $q^L$ mechanically mitigates the mechanisms, even though the economic forces and their qualitative implications remain the same.

When solving this extension, we have kept the values of other parameters from the calibration of our main model. What we can do alternatively is to recalibrate all parameters to rematch all the moments after firm type transition is introduced. Doing so will likely enlarge the quantitative effects of our mechanisms because the channels mitigated by type transition will have to be strengthened by adjusting other parameters so that the equilibrium dynamics can still match the moments. However, we choose not to fully recalibrate all parameters and keep the parameter values from the main model as the purpose of this exercise is to show transparently the role of type transition. Next we present how we choose $\tilde{\eta}$ and the solution of this extended model.

Since $\tilde{\eta}$ directly affects the persistence of TFP process $A_t \in \{A^L, A^H\}$, we will discipline the calibration of $\tilde{\eta}$ using the AR(1) coefficient of firm-level TFP process. We use the same dataset introduced in Section C.1 on firms' TFP at yearly frequency. We estimate a pooled panel regression of TFP on its one-year lag, restricting to firms with at least 5 years of observations for the reliability of our estimation. We find an AR(1) coefficient of 0.84. Our estimate is in line with the estimates of AR(1) coefficient of firms' TFP in the literature. For example, Foster, Haltiwanger, and Syverson (2008) document a range of productivity persistence from 0.76 to 0.81, and Cooper and Haltiwanger (2006) decompose productivity into a firm-specific component that has an AR(1) coefficient of 0.89 and a common component that has an AR(1) component of 0.76. Next, we calculate the model counterpart by simulating the model over 50 years (in line with the sample period of our data) across 5000 runs, and averaging the implied AR(1) coefficient across simulation runs. Matching this average model-implied AR(1) coefficient with the data counterpart, we obtain $\tilde{\eta} = 0.04$.

With the estimated $\tilde{\eta}$, we proceed to solve the full model. Note that the type transition does not change $dK_t$ given by (22). Moreover, $d\omega_t$ depends on the growth rate of $K_t^L$ and $K_t^H$, and the impact of $\tilde{\eta}$ in the two growth rates cancels out.

Next, we adjust the capital valuation equations. Since there is type transition, capital value $q^j(\omega_{t-})$ becomes intertwined between $q^H$ and $q^L$.

$$r = \mathbb{E}_t\left[\frac{dq_t^j/dt}{q_{t-}^j}\right] + \frac{A^j - \phi(\iota_{t-}^j)}{q_{t-}^j} + (\iota_{t-}^j - \delta) + \lambda\frac{\int_0^\infty \pi_t^j(\zeta)dH(\zeta) - q_{t-}^j}{q_{t-}^j} + \tilde{\eta}\frac{q_t^{-j} - q_{t-}^j}{q_{t-}^j}, \quad \text{(A41)}$$

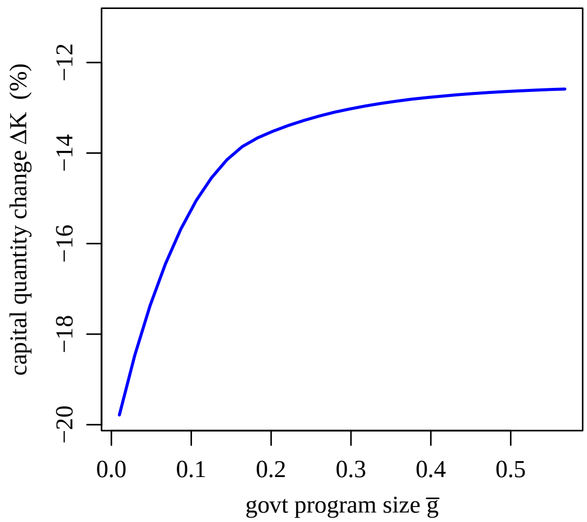
where we denote $q_t^{-j}$ as the capital value for the opposite type of $j$. This equation still leads to a constant valuation, i.e., $q_t^j = q^j$. Rewriting the valuation equation, we get

$$\begin{aligned} 0 &= A^j - \phi(\iota^j) + \iota^j q^j - (\delta + r)q^j + \tilde{\eta}(q^{-j} - q^j) \\ &\quad + \lambda\left(\int_0^\infty F(x^j(\zeta) + \zeta)\max\{q^j - (1 + r^j)x^j(\zeta), \beta\}dH(\zeta) - q^j\right). \end{aligned} \quad \text{(A42)}$$

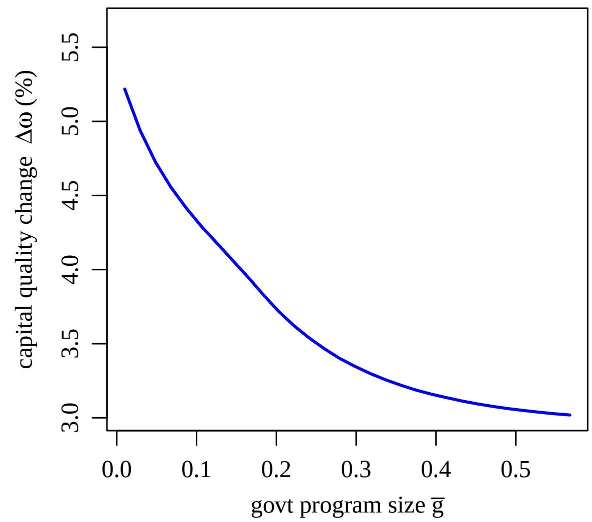
Without type transition, we have $q^H = 2.46$ and $q^L = 0.56$ in the main model. After introducing type transition, we get $q^H = 2.06$ and $q^L = 0.73$. Therefore, type transition narrow the wedge between $q^H$ and $q^L$. Mechanically, when type-$H$ can become type-$L$ with certain probability and vice versa, the capital values of two types are pulled closer.

Finally, with these modified capital valuations and state-variable dynamics, we solve the model. As discussed in the main text, the differences between type-$H$ and -$L$ firms manifest into the difference in capital values, $q^H$ and $q^L$, and the type difference shows up in firms' financing decisions in crises and decisions to repay strategically default through these capital values.

In Figure A10, we plot the impact of intervention scale $\bar{g}$ on capital quantity and quality. Introducing type transition does not change the fact that the policy maker faces the trade-off between




(a) Credit intervention and capital quantity



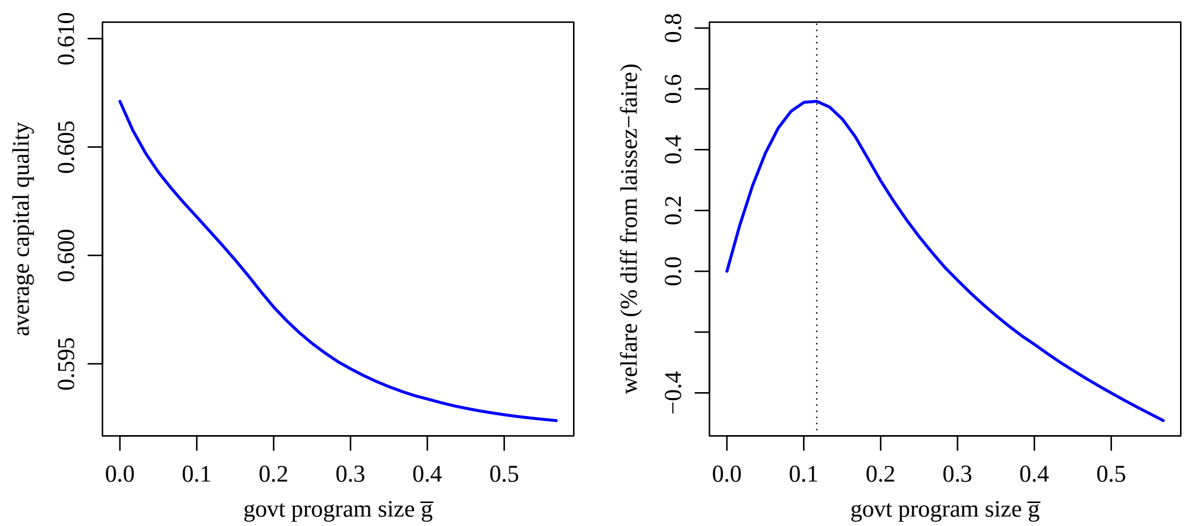

(b) Credit intervention and capital quality

**Figure A10: Credit intervention impact: Capital quantity vs. quality (with type transition).** This figure illustrates how $\bar{g}$ affects $\Delta K_t$ and $\Delta\omega_t$ in a crisis, under the alternative model with type transitions. The calculation requires the pre-crisis $\omega_{t-}$ which we set to the average value $\bar{\omega}$. This figure contrasts with Figure 2 in the main text.

quantity (panel A) and quality (panel B). With a narrower wedge between $q^H$ and $q^L$, the impact of intervention on both $K_t$ and $\omega_t$ becomes smaller, but not significantly different from Figure 2.

In Figure A11, we show how intervention affects the long-run average $\omega_t$ (calculated from the stationary distribution) and welfare under type transition. In panel A, as the two types of firms effective become more similar to one another under type transition, the impact of invention on the long-run average of $\omega_t$ is smaller than that in our main model. In panel B of Figure A11, we find that the optimal scale of intervention is similar to that in Figure 4. The reason is that with lower $q^H$ but higher $q^L$, the beneficial and detrimental aspects of intervention on welfare offset each other.

In Figure A12, we show how intervention in the current crisis affects the scale of future interventions, similar to what we have done in Figure 6 in the main text. Panel A shows that in comparison to panel A of Figure 6, the impact of varying $\bar{g}$ in the current crisis on $\omega_t$ ten years later is smaller under firm type transition. As previously discussed, type transition pulls together capital values of type-$H$ and $L$ and thus reduces the impact of intervention on firm quality. Panel B shows the impact of varying $\bar{g}$ in the current crisis on the scale of intervention should another crisis happen in ten years. As expected, the pass-through rate is smaller than that shown in panel B



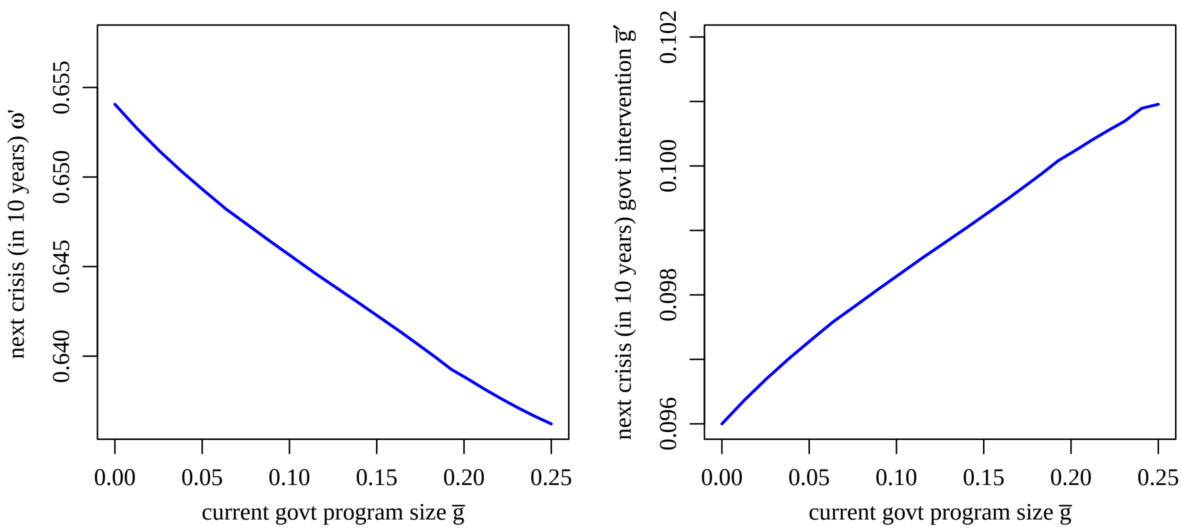

(a) Average capital quality

(b) Welfare improvement

**Figure A11: Credit intervention and welfare (with type transition).** In panel A, we plot the average firm quality $\bar{\omega}$ as a function of government intervention $\bar{g}$. For each $\bar{g}$, we solve the model again and calculate the average of simulated $\omega_t$ as $\bar{\omega}(\bar{g})$. In panel B, we show the welfare difference $W(\bar{\omega}(\bar{g}); \bar{g})/W(\bar{\omega}(0); 0) - 1$ as a function of government intervention $\bar{g}$. This figure contrasts with Figure 4 in the main text.

of Figure 6 in the main text. Note that to be comparable with panel B of Figure 6, we set the policy goal in this extended model to contain output drop at $\underline{y}$ in crises where $\underline{y}$ is set so that the starting point of the curve $\bar{g}'(\bar{g} = 0)$, i.e., the scale of next intervention under no intervention in the current crisis, is the same as starting point of the curve in panel B of Figure 6.

In summary, introducing firm type transition effectively makes the two types of firms more similar to one another, which mechanically reduces the quantitative effects of our mechanisms but does not change the qualitative dynamics. We want to emphasize that under type transition, the economic magnitude of policy distortions is still quite significant.

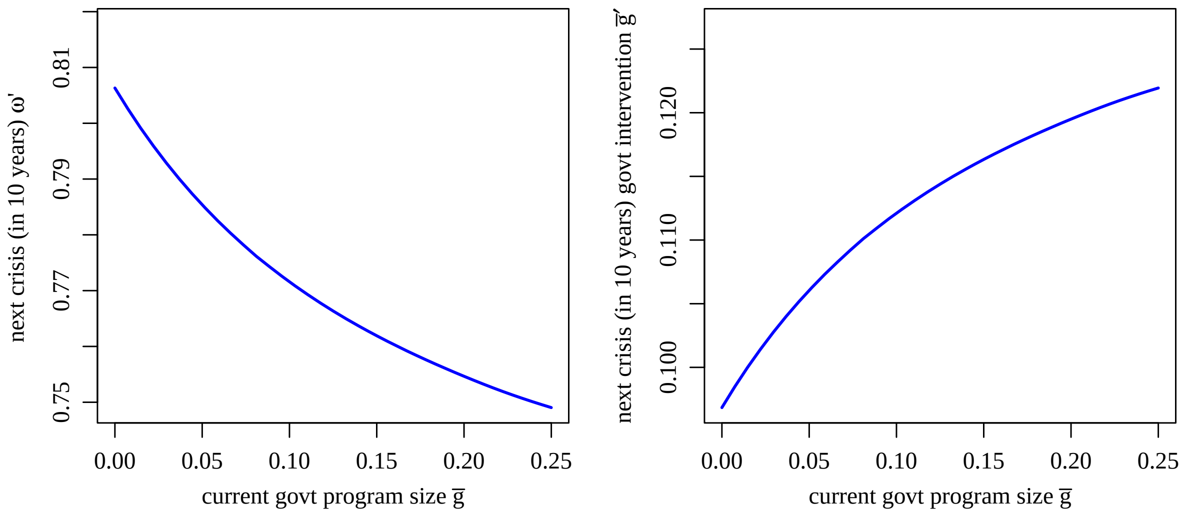



(a) Firm quality entering the next crisis

(b) Required intervention in the next crisis

**Figure A12: Intervention pass-through across crises (with type transition).** We show how $\bar{g}$ in the current crisis affects capital quality entering the next crisis, $\omega'$, and intervention needed, $\bar{g}'$, in the next crisis to contain output drop to be $\underline{y}$. This limit $\underline{y}$ is set so that the starting point of the curve in Panel B, denoted by $\bar{g}'(\bar{g} = 0)$, is the same as Figure 6 in the main text. The next crisis happens ten years after the current one. Agents expect no intervention (pass-through is only due to forward propagation). The current crisis happens at $\omega$ equal to the average value of $\omega_t$ in the laissez-faire economy.

## D.3 Alternative Policy Goals




(a) Firm quality entering the next crisis

(b) Required intervention in the next crisis

**Figure A13: Intervention pass-through across crises (alternative policy goal).** We show how $\bar{g}$ in the current crisis affects capital quality entering the next crisis, $\omega'$, and intervention needed, $\bar{g}'$, in the next crisis alternative policy goal $\Delta^K$ to be $\underline{\Delta}^K$. This $\underline{\Delta}^K$ is set so that the starting point of the curve in Panel B, denoted by $\bar{g}'(\bar{g} = 0)$, is the same as Figure 6 in the main text. The next crisis happens ten years after the current one. Agents expect no intervention (pass-through is only due to forward propagation). The current crisis happens at $\omega$ equal to the average value of $\omega_t$ in the laissez-faire economy.

We consider an alternative objective function: in a crisis, the government aims to limit the fraction of productive capital being destroyed. The motivation is that the scale of production in the economy, determined by the capital stock, drives employment. Even though we do not explicitly model the labor market, it is reasonable to expect a connection between capital and labor. Therefore, preserving the capital stock sustains employment. We demonstrate the slippery slope of intervention under this alternative policy goal. In panel A of Figure A13, we show the persistent impact of intervention $\bar{g}$ on firm quality $\omega_t$ in next 10 years. In panel B, we show that to achieve this alternative policy goal, the required scale of intervention in the next crisis, denoted by $\bar{g}'$. The government aims to limit capital quantity drop $\Delta^K$ to be $\underline{\Delta}^K$ (rather than to limit output drop) where $\underline{\Delta}^K$ is set so that the starting point of the curve in Panel B, $\bar{g}'(\bar{g} = 0)$, is the same as that in Figure 6 in the main text for comparison. We find that the slippery slope is robust and the inter-crisis pass-through is even stronger than the baseline results in Figure 6.

## D.4 Alternative Designs of Government Intervention

In our baseline model, the liquidity support takes the form of loans and follows market-based interest rates, which discourages take-up by firms that will actually repay the loans (i.e., those with $\zeta \geq \underline{\zeta}_t^j$, $j \in \{H, L\}$). Requiring loan repayment does not directly discipline firms that strategically default (i.e., those with $\zeta < \underline{\zeta}_t^j$, $j \in \{H, L\}$). Next we consider an alternative policy design, replacing the liquidity support in the form of loans by simple subsidy that does not require repayment. When the scale of subsidy is properly chosen, it improves welfare relative to the policy design in our main model (i.e., the government extending loans to firms).

This proposal seems defying the Bagehot's principle to lend freely at high rates in crises. However, Bagehot's principle requires liquidity support to be provided only to solvent firms and against good collateral. In our setting, the government cannot differentiate firms (and their productive capital) of different types. Under this realistic restriction, lending at a high rates is actually detrimental because it only limits the borrowing of firms that actually repay and does not discipline those that take liquidity support and default. The inefficiency in our model arises from firms that strategically default and over-spend in crises; liquidity for firms that repay the loans improves efficiency.

Specifically, we consider a subsidy that is up to $\bar{g}k_t^j$ for a firm with capital stock $k_t^j$. Therefore, the take-up limit is in the same format as our main model, but the government does not ask for repayment. Because this is a subsidy, the take-up rate is 100%, and $\bar{g}$ directly goes into the survival probability. Therefore, the aggregate take-up is $\bar{g}K_t$. In the following, we maintain the same notations but note that equilibrium values of the endogenous variables can differ from those in our main model as intervention is to provide subsidy not loans that charge market-based interest rates.

The private-sector creditors still charge an interest rate $r_t^j(\zeta, d)$ on a type-$j$ firm with realized $\zeta$ and private-sector debt $d$ per unit of capital. Total funds available to spend is $x = d + \bar{g}$. The firm chooses the amount of private-sector debt financing to maximize the expected value:

$$d_t^j(\zeta) = \arg\max_{0 \leq d \leq \bar{d}} F(d + \bar{g} + \zeta) \left[ q_t^j - (1 + r_t^j(\zeta, d))d \right] , \tag{A43}$$

where the interest rate, $r_t^j(\zeta, d)$, is set by the creditors' break-even condition as in our main model:

$$F(d + \bar{g} + \zeta)(1 + r_t^j(\zeta, d))d = d\,. \tag{A44}$$

Note that $\bar{\zeta}_t^j$ is defined in the same way as in the main model by equation (14). Under the subsidy and before introducing the incentive to strategically default, the firm chooses

$$d_t^{*j}(\zeta) = (\bar{\zeta}_t^j - \bar{g} - \zeta)^+, \tag{A45}$$

A firm's optimal level of private-sector financing is the minimum of this first-best (socially optimal) level and the available funding from the private-sector creditors, i.e., $d_t^j(\zeta) = \min\{d_t^{*j}(\zeta),\, \bar{d}\}$.

Next, we introduce strategic default. Let $\underline{\zeta}_t^j$ denote the solution to the following indifference condition over $\zeta$ as in the main text,

$$q_t^j - \left[1 + r_t^j\left(\underline{\zeta}_t^j, d_t^j(\underline{\zeta}_t^j)\right)\right] d_t^j(\underline{\zeta}_t^j) = \beta. \tag{A46}$$

When $\zeta < \underline{\zeta}_t^j$, the firm chooses default. When $\zeta \geq \underline{\zeta}_t^j$, the firm chooses repaying private-sector debt and continuing operations. As in our main model, beyond the interest rate $r_t^j\left(\zeta, x_t^j(\zeta)\right)$, the private-sector creditors also specify a debt limit for firms with $\zeta < \underline{\zeta}_t^j$, denoted by $\hat{d}_t^j(\zeta)$,

$$\hat{d}_t^j(\zeta) = F(\hat{d}_t^j(\zeta) + \zeta + \bar{g})(q_t^j - \beta)\,, \tag{A47}$$

so that the private-sector creditors break even (through recovery value) when lending to these firms.

In summary, a firm's optimal strategy for private-sector borrowing is

$$d_t^j(\zeta) = \underbrace{\mathbf{1}_{\zeta \geq \underline{\zeta}_t^j} \min\{(\bar{\zeta}_t^j - \bar{g} - \zeta)^+, \bar{d}\}}_{\text{no strategic default}} + \underbrace{\mathbf{1}_{\zeta < \underline{\zeta}_t^j}\left(\min\{\hat{d}_t^j(\zeta), \bar{d}\}\right)}_{\text{strategic default}}\,. \tag{A48}$$

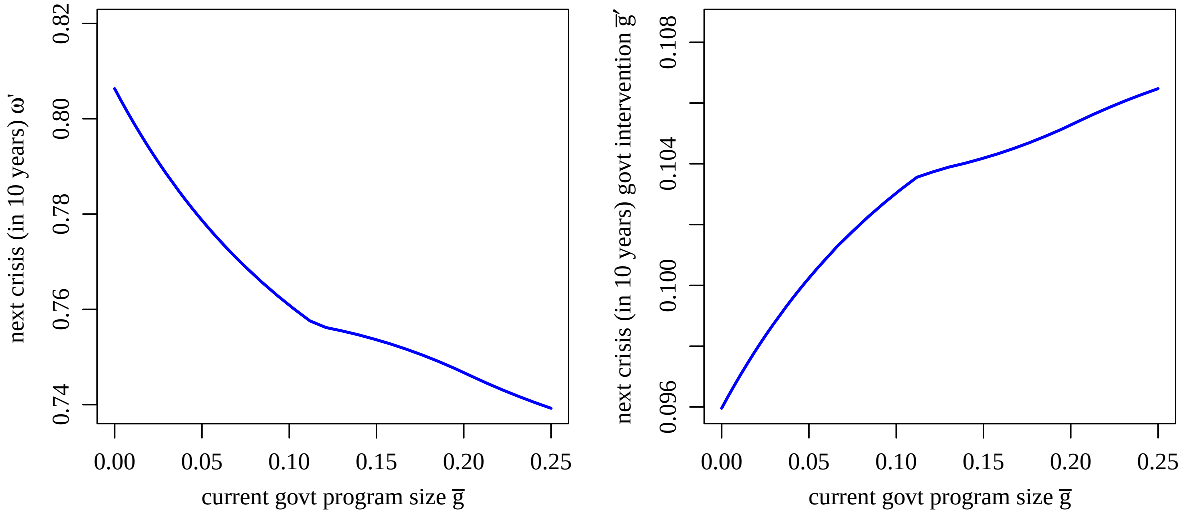
The optimal total spending on improving survival probability for a type-$j$ firm with realized $\zeta$ is

$$x_t^j(\zeta) = d_t^j(\zeta) + \bar{g} \tag{A49}$$

The expected value per unit of type-$j$ capital to the owner of a type-$j$ firm with a realized $\zeta$ is

$$\pi_t^j(\zeta) = \underbrace{\left(F(d_t^j(\zeta) + \bar{g} + \zeta)q_t^j - d_t^j(\zeta)\right)\mathbf{1}_{\zeta \geq \underline{\zeta}_t^j}}_{\text{no strategic default}} + \underbrace{F(d_t^j(\zeta) + \bar{g} + \zeta)\beta\mathbf{1}_{\zeta < \underline{\zeta}_t^j}}_{\text{strategic default}}\,. \tag{A50}$$

Finally, we solve $q^H$ and $q^L$ using the capital valuation equation (21) but a different expected value $\pi_t^j(\zeta)$ from (A50) and a modified total spending on survival in crises given by (A49).




(a) Firm quality entering the next crisis

(b) Required intervention in the next crisis

**Figure A14: Intervention pass-through across crises (pure government subsidy).** We show how $\bar{g}$ in the current crisis (government intervention is pure subsidy) affects capital quality entering the next crisis, $\omega'$, and intervention needed, $\bar{g}'$, in the next crisis alternative policy goal $\Delta^K$ to be $\underline{\Delta}^K$. This $\underline{\Delta}^K$ is set so that the starting point of the curve in Panel B, denoted by $\bar{g}'(\bar{g} = 0)$, is the same as Figure 6 in the main text. The next crisis happens ten years after the current one. Agents expect no intervention (pass-through is only due to forward propagation). The current crisis happens at $\omega$ equal to the average value of $\omega_t$ in the laissez-faire economy.

In Figure A14, we show that the slippery slope of government intervention is still present under this new policy design. Panel A shows that more government intervention in the current crisis significantly reduces firm quality even after 10 years. This persistent effect on firm quality necessitates larger government intervention in the next crisis, as shown in Panel B. The pass-through, $\bar{g}'(\bar{g})$, is similar to that in Figure 6, indicating that quantitative significance of the slippery slope is similar, although the new policy design is more efficient as discussed in Section 3.4.

## D.5 Extension: Different Types of Liquidity Crises

In our baseline model in the main text, the liquidity crisis involves spending real resources. This is a standard assumption in the literature. For example, in Holmström and Tirole (1998), when a liquidity shock hits, a firm needs to make real investment to protect its project, and such spending cannot be recovered after the crisis. This assumption is also supported by reality: during the Covid-19 pandemic, many businesses have to adapt their products, incurring real costs, to protect their franchise value including but not limited to customer capital and organizational capital.

However, there are also scenarios in which crisis spending does not involve using up real resources. Consider a firm that has account receivables due from its customers. In a liquidity crisis, customers' payment is delayed, so the firm must raise external financing to cover the wages owed to workers, the rent owed to landlords, and other operating expenses. After the crisis, the firm is able to collect at least part of the account receivables and obtain the cash due from customers. External financing in this liquidity crisis is just to bridge the firm through the crisis.

The question is to what extent the liquidity crisis entails spending on real resources as in Holmström and Tirole (1998) and to what extent the crisis is about situations like the example above on delayed payment and recoverable cash. The reality is somewhere in between. Thus, we extend the model by introducing a parameter, $\kappa_x$ ($\in (0,1)$), for the fraction of spending on surviving a crisis that cannot be recovered after the crisis. In our baseline model, $\kappa_x = 1$. In the following, we present and analyze the extended model. We show that all of our theoretical (qualitative) results carry through and provide provide quantitative results under different values of $\kappa_x$.

We assume that a percentage $\kappa_x$ of spending on improving survival probability is actually spent on the real resources (generic goods), while the $1-\kappa_x$ fraction can be recovered after the firm survives the crisis. The recovered cash $1-\kappa_x$ will accrue to the firm owners if the firm survives the crisis, but to creditors if the firm fails to survive and the creditors seize the firm. Without any frictions and incentive distortions (e.g., strategic default), the firm's problem is given by

$$\max_x F(x+\zeta)\left[q_t^j - (1+r_t^j(\zeta,x))x + (1-\kappa_x)x\right], \tag{A51}$$

which is the extended version of equation (12) in the main text. The social planner's problem is

$$\max_x F(x+\zeta)q_t^j + (1-\kappa_x)x - x, \tag{A52}$$

which extend the equation (10) in the main text.

Solving for (A52) leads to the first-order condition

$$F'(x+\zeta)q_t^j = \kappa_x. \tag{A53}$$

Then we define $\bar{\zeta}_t^j$ as the solution to the following equation,

$$F'(\bar{\zeta}_t^j)q_t^j = \kappa_x. \tag{A54}$$

The first-best level of financing for spending in the crisis is given by

$$x_t^{*j}(\zeta) = (\bar{\zeta}_t^j - \zeta)^+, \tag{A55}$$

which is in the same functional form as our main setup and thus proves Lemma 1.

To solve for the firm's decision problem in (A51), we first derive the creditor break-even condition,

$$F(x+\zeta)(1+r_t^j(\zeta,x))x + (1-F(x+\zeta))(1-\kappa_x)x = x. \tag{A56}$$

Substituting out $r_t^j(\zeta,x)$ in the firm's objective function using this break-even condition (A56), we obtain exactly the same objective as the social planner's. Thus, in the case without frictions and incentive distortions, the firm's decision coincides with the social planner's.

After we allow for strategic default, the firm compares the value under repayment, $q_t^j - (1 + r_t^j(\zeta,x))x_t^j(\zeta) + (1-\kappa_x)x_t^j(\zeta)$, with the value from strategic default, $\beta + (1-\kappa_x)x_t^j(\zeta)$, which includes obtaining $\beta$ as in the original setup and absconding the recovered cash. The threshold $\underline{\zeta}_t^j$ that makes the firm indifferent between the two is given by

$$q_t^j - (1 + r_t^j(\underline{\zeta}_t^j, x))x_t^j(\underline{\zeta}_t^j) = \beta, \tag{A57}$$

which is the same as equation (17) of our main text. This leads to the same properties of $\underline{\zeta}_t^j$ as in the main setup. Therefore, we can prove Lemma 2 under this extended setting as well.

Next, the equation for determining the endogenous lending threshold for creditor is

$$\hat{d}_t^j(\zeta) = F(\hat{d}_t^j(\zeta) + \zeta + \bar{g})\left[(q_t^j + (1-\kappa_x)x_t^j) - (\beta + (1-\kappa_x)x_t^j)\right] \tag{A58}$$

which can be simplified to

$$\hat{d}_t^j(\zeta) = F(\hat{d}_t^j(\zeta) + \zeta + \bar{g})(q_t^j - \beta) \tag{A59}$$

This equation that defines $\hat{d}_t^j(\zeta)$ is exactly the same as equation (18) of our main text.

Finally, the firm's optimal choice of $x$ can be summarized as follows

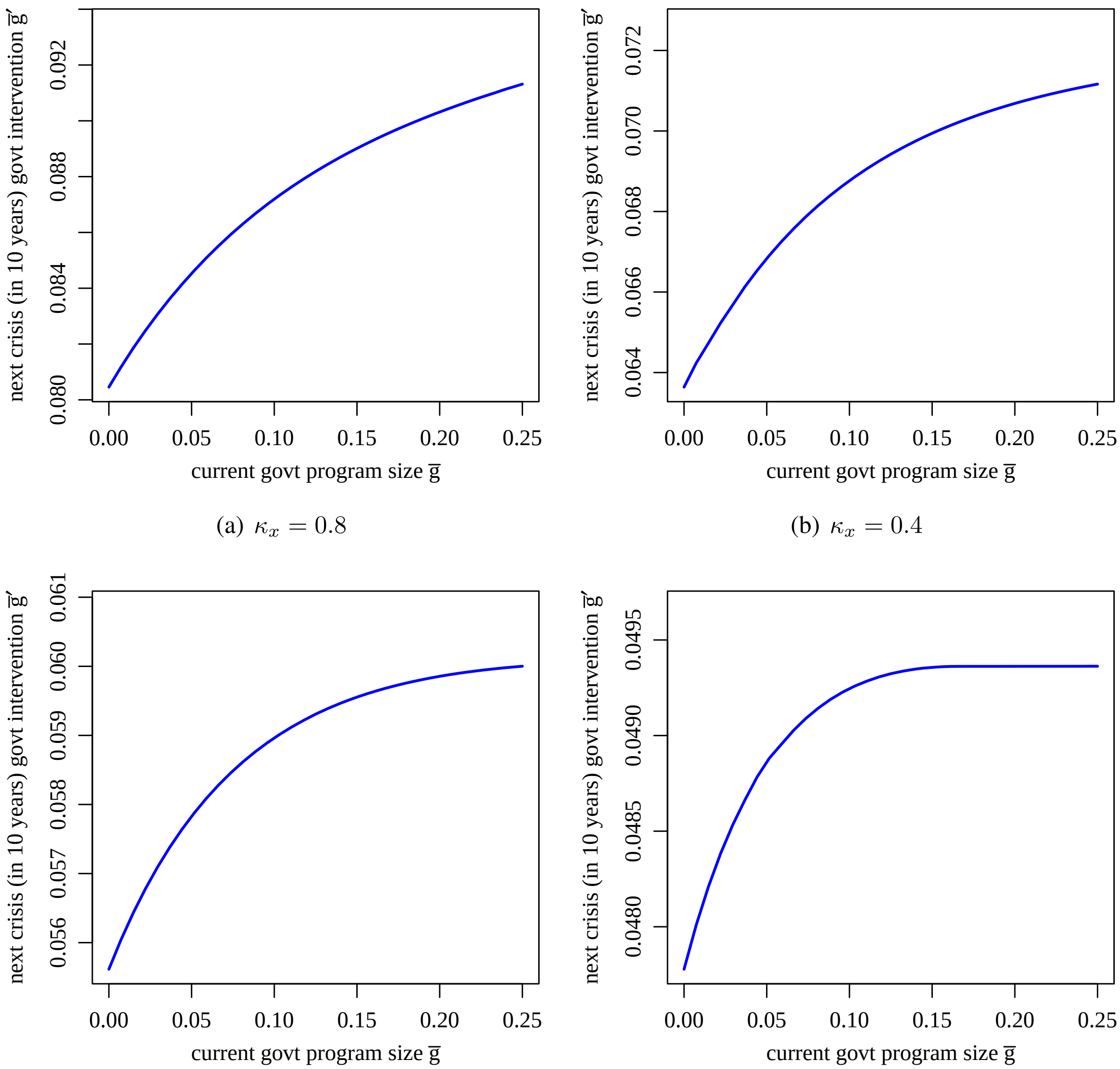
$$x_t^j(\zeta) = \mathbf{1}_{\zeta \geq \underline{\zeta}_t^j} \min\{(\bar{\zeta}_t^j - \zeta)^+, \bar{d} + \bar{g}\} + \mathbf{1}_{\zeta < \underline{\zeta}_t^j} \left(\min\{\hat{d}_t^j(\zeta), \bar{d}\} + \bar{g}\right) \tag{A60}$$

which is the same as the equation (19) in our main text. The impact of $\kappa_x$ is absorbed in $\bar{\zeta}_t^j$. Given the identical functional form, we can prove Proposition 1 similarly. Moreover, since $\kappa_x$ only affects $\bar{\zeta}_t^j$ without changing any other decision rules given the value of $\bar{\zeta}_t^j$, we find that Proposition 2 also holds. Since all other equilibrium conditions remain the same, we can show that Proposition 3 to 7 all hold. In summary, our theoretical results carry through in the extended setting.

In the following, we fix the other parameters and present the quantitative results on the slippery slope of policy intervention under $\kappa_x = 0.8, 0.4, 0.2$ and $0.1$ (i.e., we show the comparative statics). As previously discussed, the case of $\kappa_x = 1$ corresponds to our baseline model. The lower $\kappa_x$ is, the further away the alternative model is from our baseline model. Our main finding is that as $\kappa_x$ decreases from one, the quantitative magnitude of the slippery slope remains robust and significant for a large range of values, and then the magnitude starts to decline when $\kappa_x$ becomes very low. This finding has an interesting implication: our mechanism is more potent in crises when firms must spend real resources to survive as in Holmström and Tirole (1998) than in crises where firms only need bridge loans to cover temporary spending needs.

In Figure A15, we illustrate the same exercise as we do in Figure 6(b) in the main text under different values of $\kappa_x$. The slippery slope is quantitatively relevant in all cases, with an intervention pass-through (the average slope in each figure) of 4.8% (panel a), 3.2% (panel b), 1.7% (panel c), and 0.6% (panel d), respectively, for $\kappa_x$ equal to $0.8$, $0.4$, $0.2$, and $0.1$. Moreover, the slope is positive across all scenarios, indicating the robustness of our qualitative results. The overall pattern is that, as we decrease the fraction of crisis spending that cannot be recovered, the intervention pass-through from the current crisis to the next crisis in ten years becomes weaker.

In Figure A16, we conduct the same exercise as we do Figure 7(b) in the main text. The peak of each panel, which can be interpreted as a measure of the strength of slippery slope (see the discussion in Section 3.3), ranges from 15% (panel a) to 14% (panel b), 10% (panel c), and 4.5% (panel d), respectively, for $\kappa_x$ equal to $0.8$, $0.4$, $0.2$, and $0.1$. Similar to the pattern indicated by Figure A15, the strength of the slippery slope decreases as we allow for a higher fraction of crisis spending to be recovered after firms survive the crisis. Across all panels, the shapes of curves are similar, indicating the robustness of our mechanism.



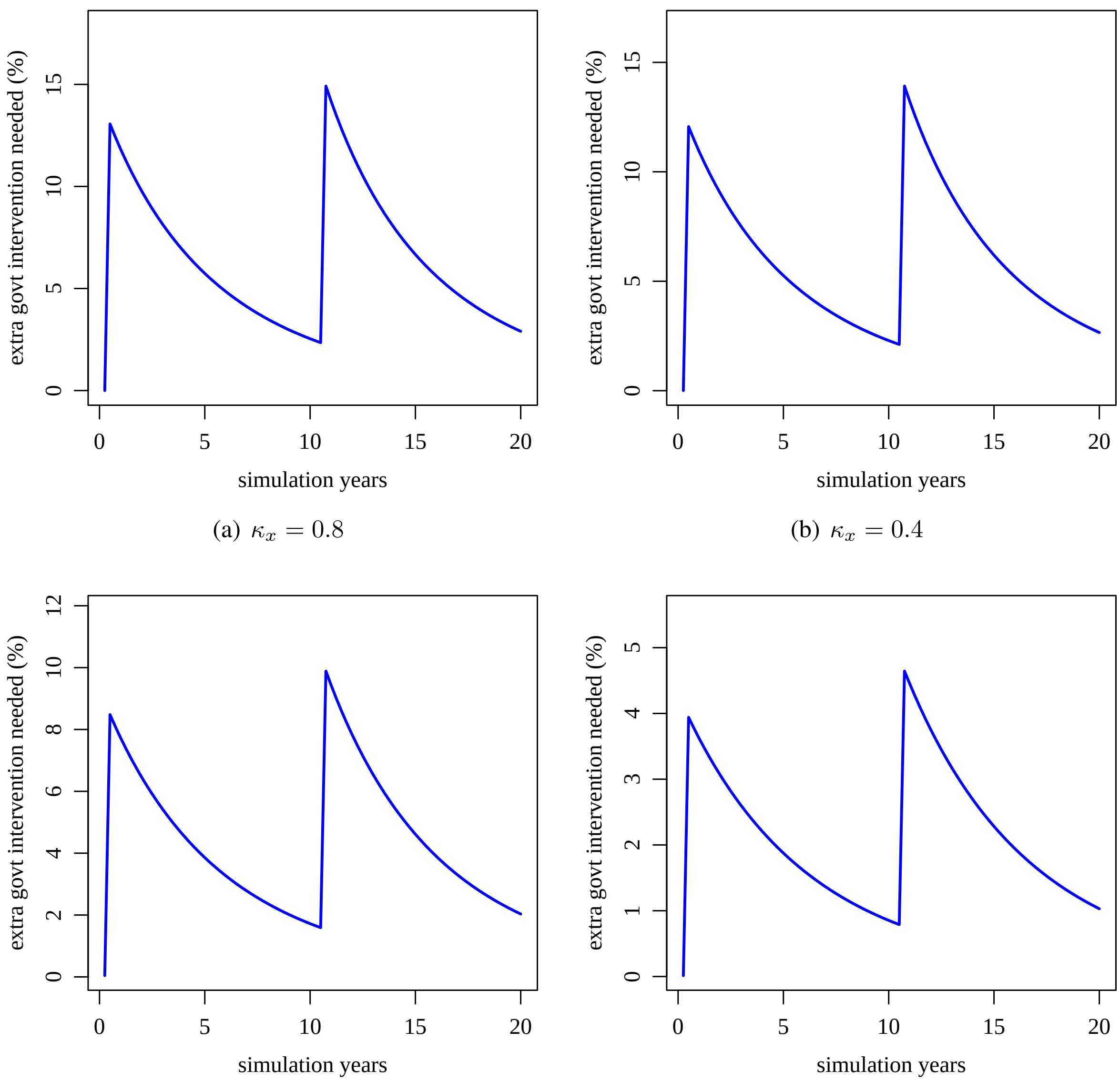

(a) $\kappa_x = 0.8$

(b) $\kappa_x = 0.4$

(c) $\kappa_x = 0.2$

(d) $\kappa_x = 0.1$

**Figure A15: Intervention pass-through across crises, with different $\kappa_x$.** We show how $\bar{g}$ in the current crisis affects intervention needed, $\bar{g}'$, in the next crisis to contain output drop within $-10\%$. The next crisis happens ten years after the current one. Agents expect no intervention (pass-through is only due to forward propagation). This figure corresponds to Figure 6(b).




(a) $\kappa_x = 0.8$

(b) $\kappa_x = 0.4$

(c) $\kappa_x = 0.2$

(d) $\kappa_x = 0.1$

**Figure A16: The slippery slope of intervention under simulated paths, with different $\kappa_x$.** We compare the quarterly simulation of two economies, the laissez-faire economy and the intervened economy without agents expectation of intervention, where one crisis happens in Q1 of the first year and another crisis in Q1 of the tenth year. Both simulations start at the average $\omega_t$ in the laissez-faire economy. For both economies, we calculate the amount of intervention required to contain output drop within $-10\%$ if a crisis happens over the next instant. Then we plot the percentage increase from the laissez-faire economy to the intervened economy. This figure corresponds to Figure 7(b).